P-PBL

What is Mathematics?

Teachers' Experiences of High-Quality Mathematics Instruction

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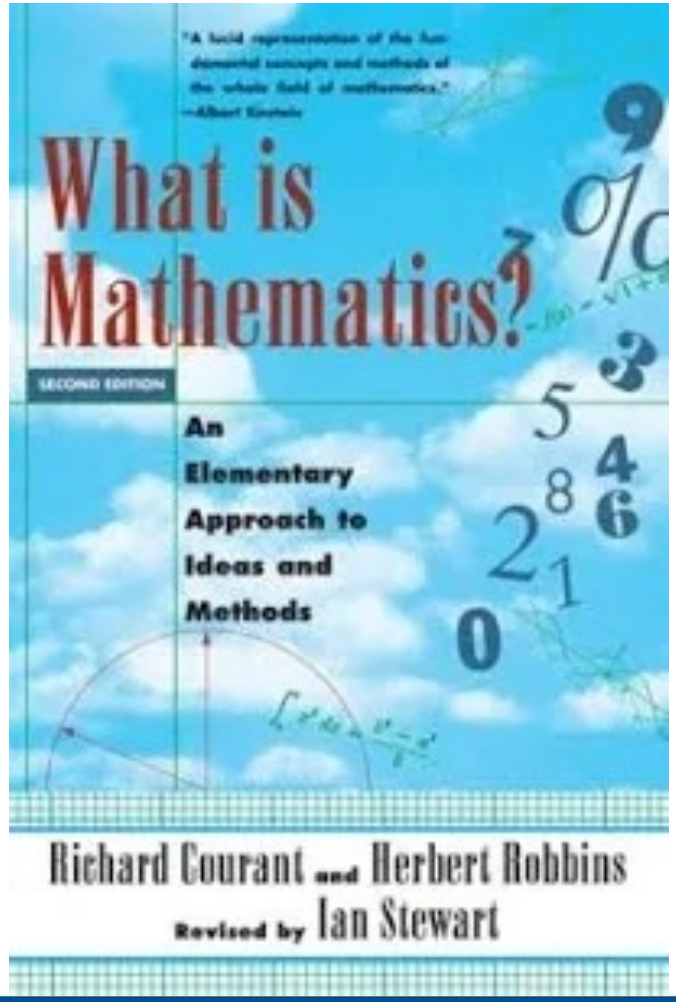
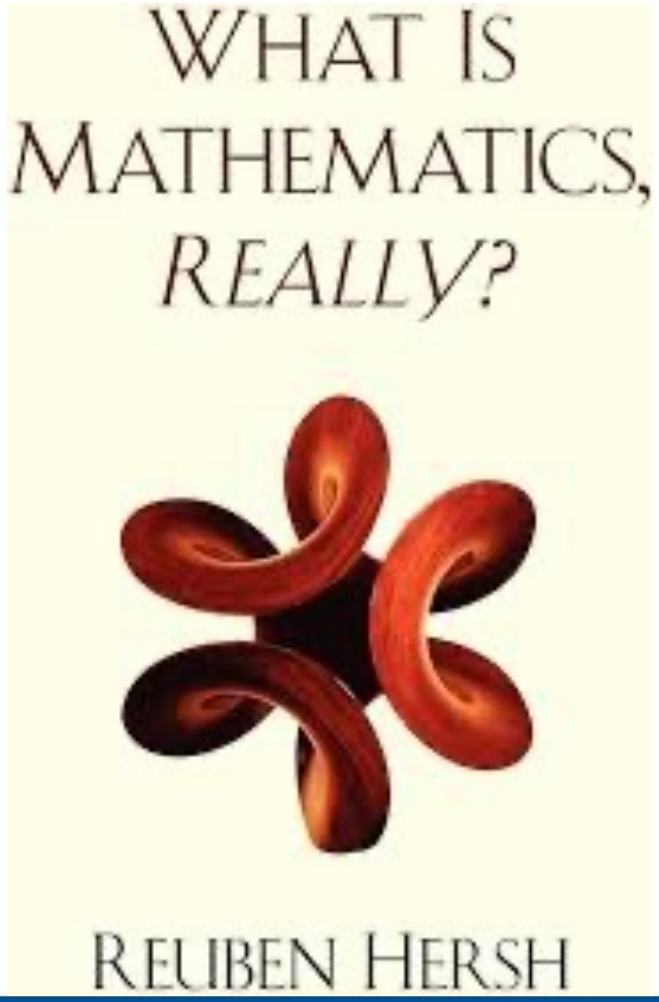
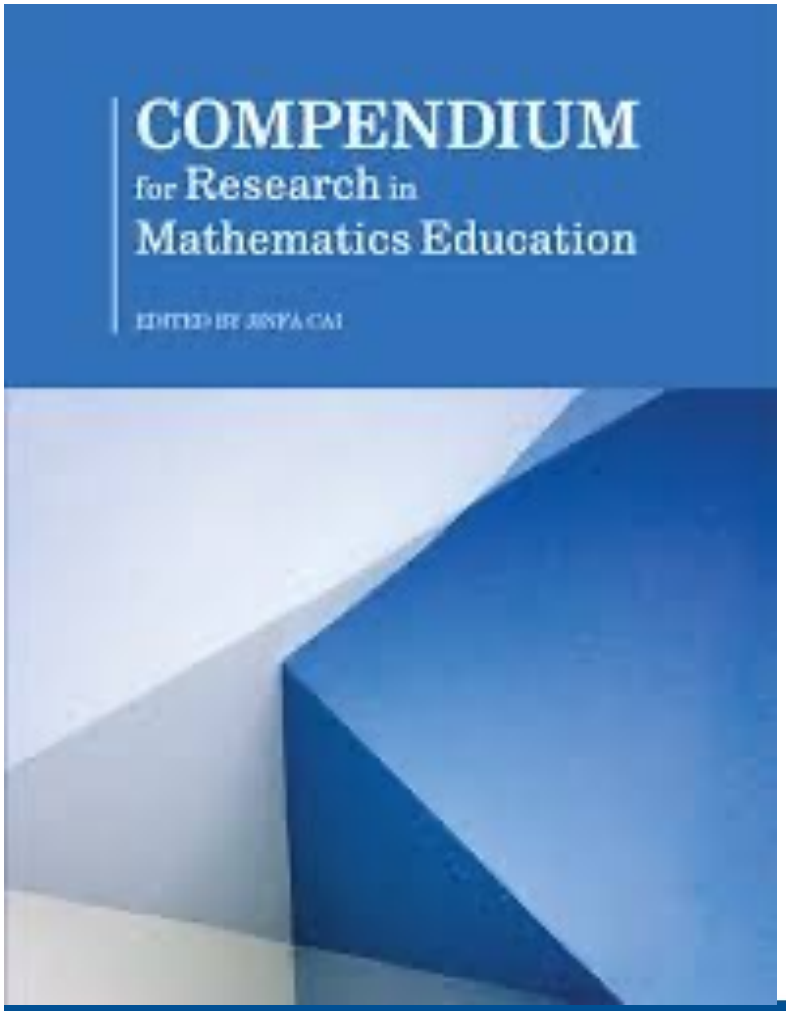
Congratulations to Jim Lewis

Thanks to Yvonne Lai
and the entire organizing team


What is Mathematics?
SECOND EDITION
An Elementary Approach to Ideas and Methods
Richard Courant and Herbert Robbins
Revised by Ian Stewart

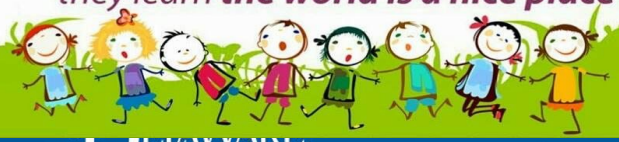

WHAT IS MATHEMATICS, REALLY?
REUBEN HERSH



Jacobs, V. R. & Spangler, D. A. (2017). Core practices in K-12 mathematics teaching. In J. Cai (Ed). Compendium of research in mathematics education (pp. 186-221). Reston, VA: National Council of Teachers of Mathematics.

Children Learn What They Live

*If children live with **criticism**, they learn to condemn.*
*If children live with **hostility**, they learn to fight.*
*If children live with **fear**, they learn to be apprehensive.*
*If children live with **pity**, they learn to feel sorry for themselves.*
*If children live with **ridicule**, they learn to feel shy.*
*If children live with **jealousy**, they learn to feel envy.*
*If children live with **shame**, they learn to feel guilty.*
*If children live with **encouragement**, they learn confidence.*
*If children live with **tolerance**, they learn patience.*
*If children live with **praise,** they learn appreciation.*
*If children live with **acceptance**, they learn to love.*
*If children live with **approval**, they learn to like themselves.*
*If children live with **recognition**, they learn it is good to have a goal.*
*If children live with **sharing**, they learn generosity.*
*If children live with **honesty**, they learn truthfulness.*
*If children live with **fairness**, they learn justice.*
*If children live with **kindness and consideration**, they learn respect.*
*If children live with **security**, they learn to have faith in themselves and in those about them.*
*If children live with **friendliness**, they learn **the world is a nice place in which to live.***



Dorothy Law Nolte *Children Learn What They Live*

Dorothy Law Nolte *Children Learn What They Live*

Dan Lortie, Schoolteacher : A Sociological Study

P-PBL

MATH 245 - AN INTRODUCTION TO PROOF

What is the sum of

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots + \frac{1}{n(n+1)} \; ?$$

Let $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots + \frac{1}{n(n+1)}$

$S_1 = \frac{1}{2}$

$S_2 = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

$S_3 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} = \frac{3}{4}$

$S_4 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} = \frac{4}{5}$

Case 1

- Jake and Anna are sitting in a library.
- Jake: I've heard from TV that it might be raining today.
- Anna: It must be raining right now. If it were not raining, then people coming into the library would be dry, but they are wet.

Case 2

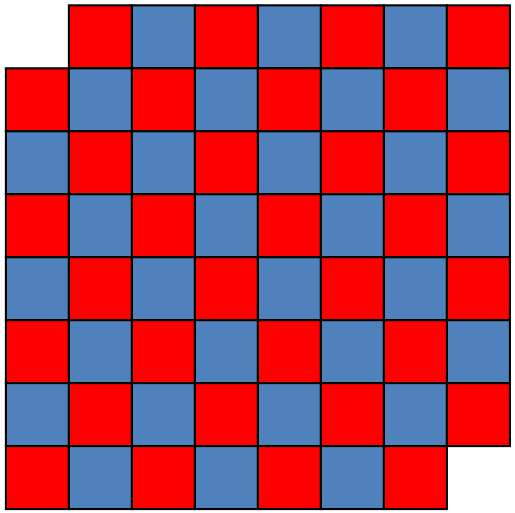
- John and Anaya drove past a football stadium.
- John: Is there any game in this stadium right now?
- Anaya: I don't think so. If a game were being played right now, the parking lots would be full of cars, but there are no cars in the lots.

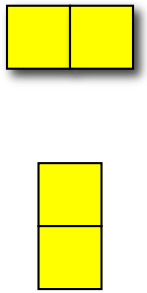
Case 3

- *A woman with her three children met a man at an apartment building.*
- *Woman: "Hi, have not seen you for a long time, how are you?"*
- *Man: "Good to see you. That's right! Have not seen you for a while. Look, your children have grown up so quickly. How old are they now?"*
- *Woman: "The product of their ages is 36."*
- *Man: "Come on, that does not give me enough information to know their ages."*
- *Woman: "The sum of their ages is the same as your apartment number."*
- *Man: "That still does not give me enough information."*
- *Woman: "My oldest child has red hair."*
- *What are the ages of this woman's three children?*

Case 4

If the two opposite corners are removed can the board be covered with dominoes? Justify your answer.





Case 5

Let $a_1, a_2, a_3, a_4, \ldots a_{(2n+1)}$ be an arrangement of the numbers 1, 2, 3, 4, … (2n+1). Show that the product of $(1- a_1)\ (2- a_2)\ (3- a_3)\ (4- a_4)\ \ldots\ [(2n+1) - a_{(2n+1)}]$ is even.

Case 6

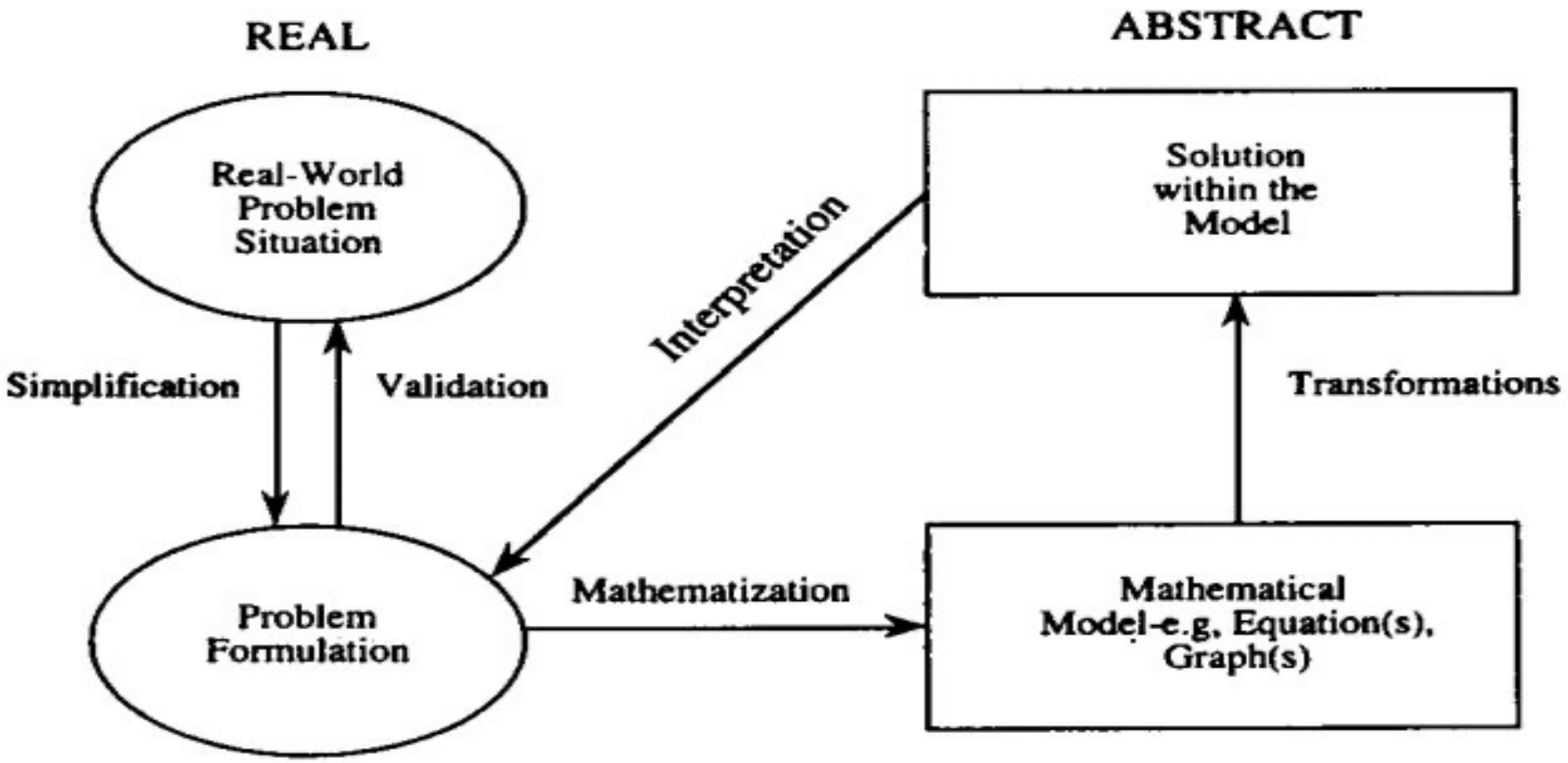
Show $\sqrt{2}$ is an irrational number.

Proof by Contrapositive (indirect proof)

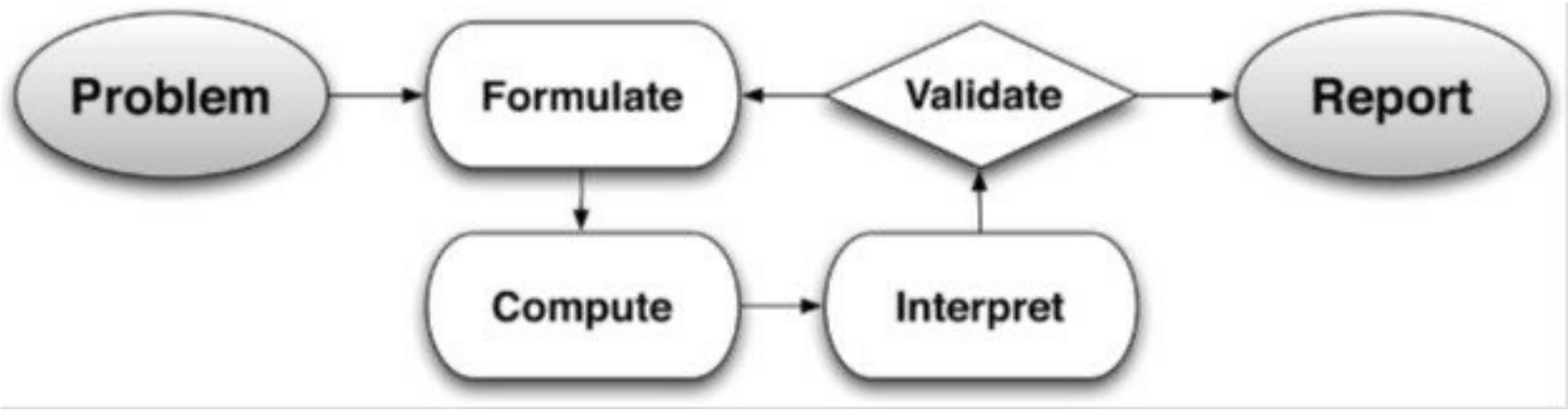
- Recall that $(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)$
- This is the basis for the proof by contraposition
  - You assume that the conclusion is false, then give a series of implications to show that such an assumption implies that the premise is false

MATH 518 - MATHEMATICAL MODELS AND APPLICATIONS

NCTM 1989


REAL
ABSTRACT
Real-World Problem Situation
Solution within the Model
Interpretation
Simplification
Validation
Transformations
Problem Formulation
Mathematization
Mathematical Model-e.g. Equation(s), Graph(s)

CCSSM, 2010



Level 1: Drug Dosage Problem

A student strained her knee in an intramural volleyball game, and her doctor has prescribed an anti-inflammatory drug to reduce the swelling. She is to take two 220-mg tablets every 8 hours for a year. Her kidneys eliminate 60% of this drug from her body every 8 hours.

Assume that she faithfully takes the correct dosage at the prescribed regular intervals.

How much drug does she have in her body after 7 days?

Level 2: Drug Dosage Problem

A student strained her knee in an intramural volleyball game, and her doctor has prescribed an anti-inflammatory drug to reduce the swelling.

How do we develop a mathematical model to predict the percentage of drug in her body as a function of time?

Level 3: Drug Dosage Problem

A student strained her knee in an intramural volleyball game, and her doctor has prescribed an anti-inflammatory drug to reduce the swelling.

Formulate an interesting and challenging mathematical modeling problem about the situation and solve the problem.

Level 4: Drug Dosage Problem

A student strained her knee in an intramural volleyball game and came to see a doctor.

Pose an easy mathematical problem and a challenging mathematical problem based on the situation and solve the problems

Students' Reactions

- I think [problem posing] gives a new perspective. It makes you think about it in a different way, makes you more familiar with the mathematics principles behind the questions.
- You're looking at the entirety of the information given, whereas if you're given a question, you're only going to give it a quick glance for what's relevant to what it's asking. You might not understand the big picture of how the problem works. Whereas if we [pose problems], you're sort of seeing the entirety of it.

Silber, 2023

An Analysis of Modeling Tasks

- *Educational Studies in Mathematics*, 2022, 109 (2), February 2022. Innovations in measuring and fostering modelling competencies (by Gabriele Kaiser & Stanislaw Schukajlow)
- ZDM – Mathematics Education, 2018, 50(1-2), Empirical research on the teaching and learning of mathematical modeling (by Gabriele Kaiser Marcelo C. Borba, Stanislaw Schukajlow, & Gloria A. Stillman
- Book Series on International Perspectives on the Teaching and Learning of Mathematical Modelling (by Gabriele Kaiser)

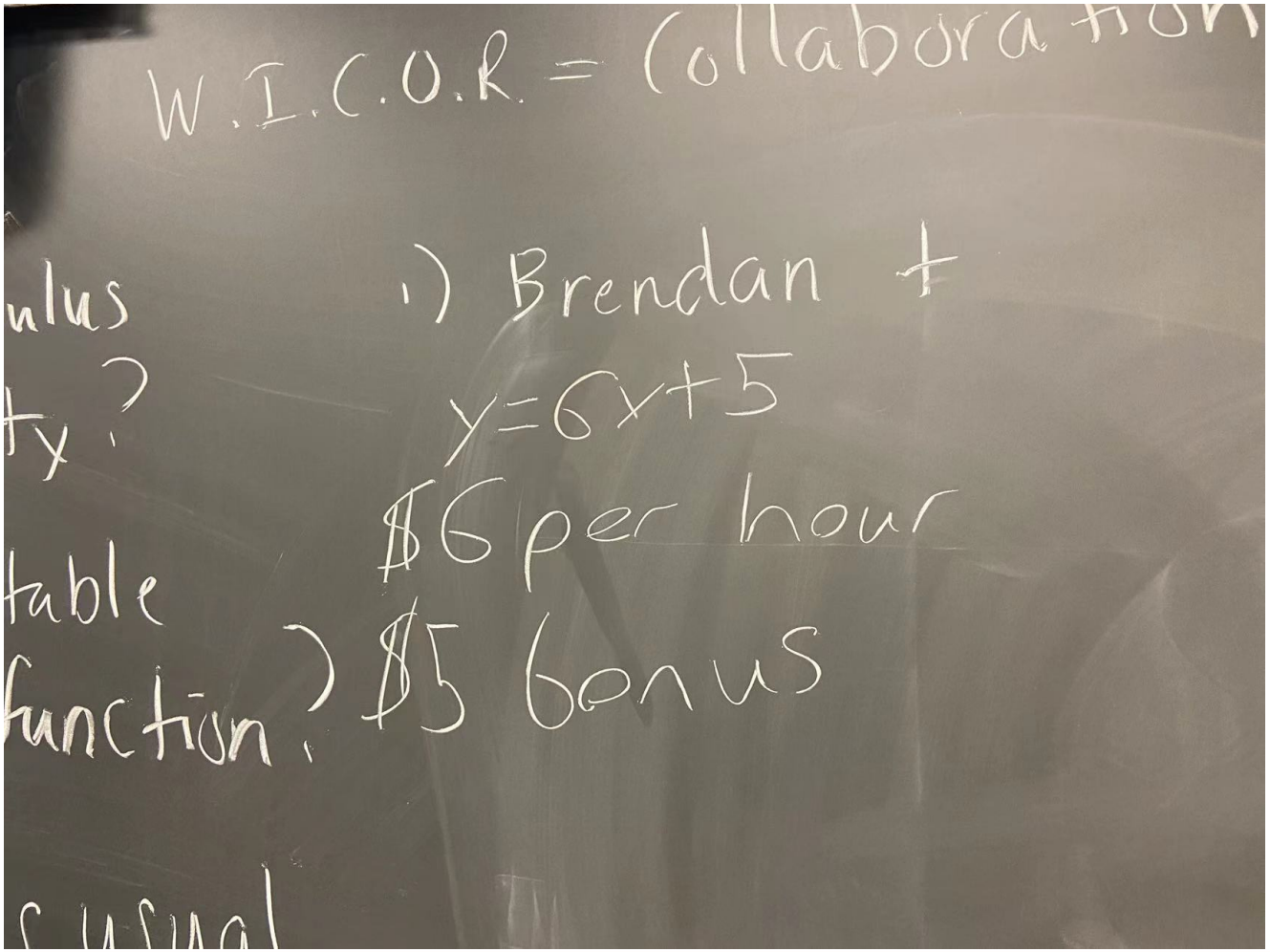
An Analysis of Modeling Tasks

- There are a total of 213 modeling tasks included in ESM Special Issue, ZDM Special Issue, and the nine books in the book series (about 450 papers).
- 12 of these 213 modeling tasks required explicitly problem formulation, which is about (less than 6%)

For the linear Function y = 6x + 5, list two ordered pairs.

"The lesson did not go all that well. While most of the students were able to plot points, many had no idea why the "order" in ordered-pair matters. They had never seen quadrants, and many did not understand how to answer the question regarding the signs of the ordered pairs in the quadrants and no student realized that there is a zero involved if the point is on an axis. The idea of producing ordered pairs for a function was beyond the grasp of all but a few of the students. MUCH WORK TO DO!"

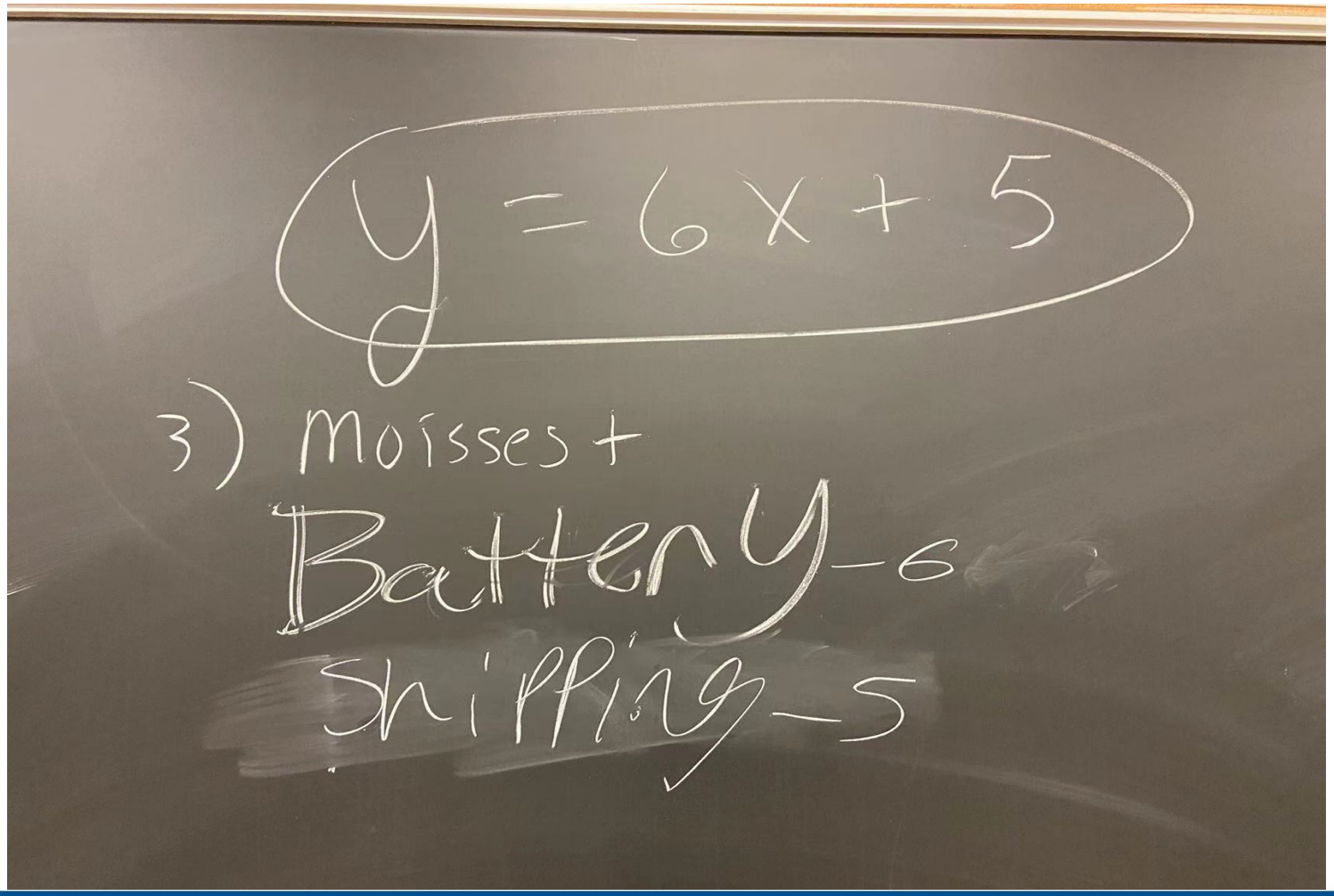
Given the linear function, y = 6x + 5, what could this represent a real-life scenario?

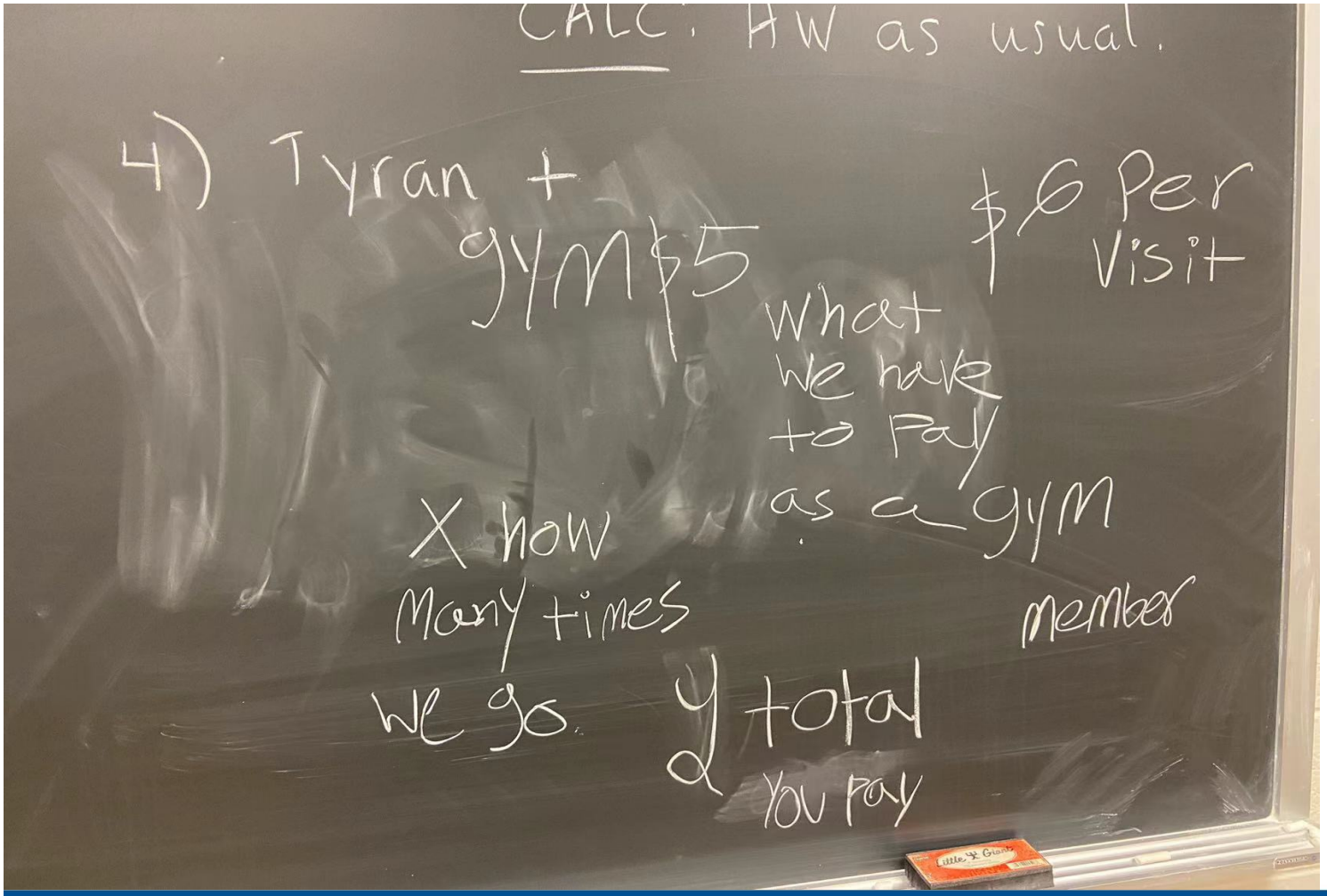


$Y = 6x + 5$

$6 per hour

$5 bonus


y = 6x + 5
3) Moisses +
Battery - 6
Shipping - 5



Gym, $5
What we have to pay as a gym member

$6 per visit

X: how many times you go

Y: total you pay

Thank you

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