

Advancing Young People's Mathematical Flourishing: Disrupting False Dichotomies

Deborah Loewenberg Ball

 @deborah_ball

JimFest: A conference in honor of W. James Lewis
May 30, 2024 • University of Nebraska–Lincoln • Lincoln, NE





In honor of Jim Lewis, an argument, complete with premises, definitions, claims, two theorems, a couple of lemmas, and evidence:

1. Fostering young people's mathematical flourishing is essential.
2. Many efforts have been made over decades to do this.
3. Still not a lot has changed inside mathematics classrooms, achievement is flat, and deep racial inequities persist.
4. False dichotomies impede efforts to advance young people's mathematics flourishing.
5. A failure to disrupt these and to understand the work of teaching is a foundational flaw.
6. Focusing on that work is a critical boundary-crossing agenda that depends on multiple forms of knowledge, humility, curiosity, respect, and nuance.



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What are we after?



building the STEM
workforce

disrupting persistent patterns of
racism that push out Black and
Brown students (and other
historically marginalized groups)

What are we after?

growing a generation
of adults who **don't**
disidentify with math

broadening our conceptions
of mathematics and
mathematical competence



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What are we after?

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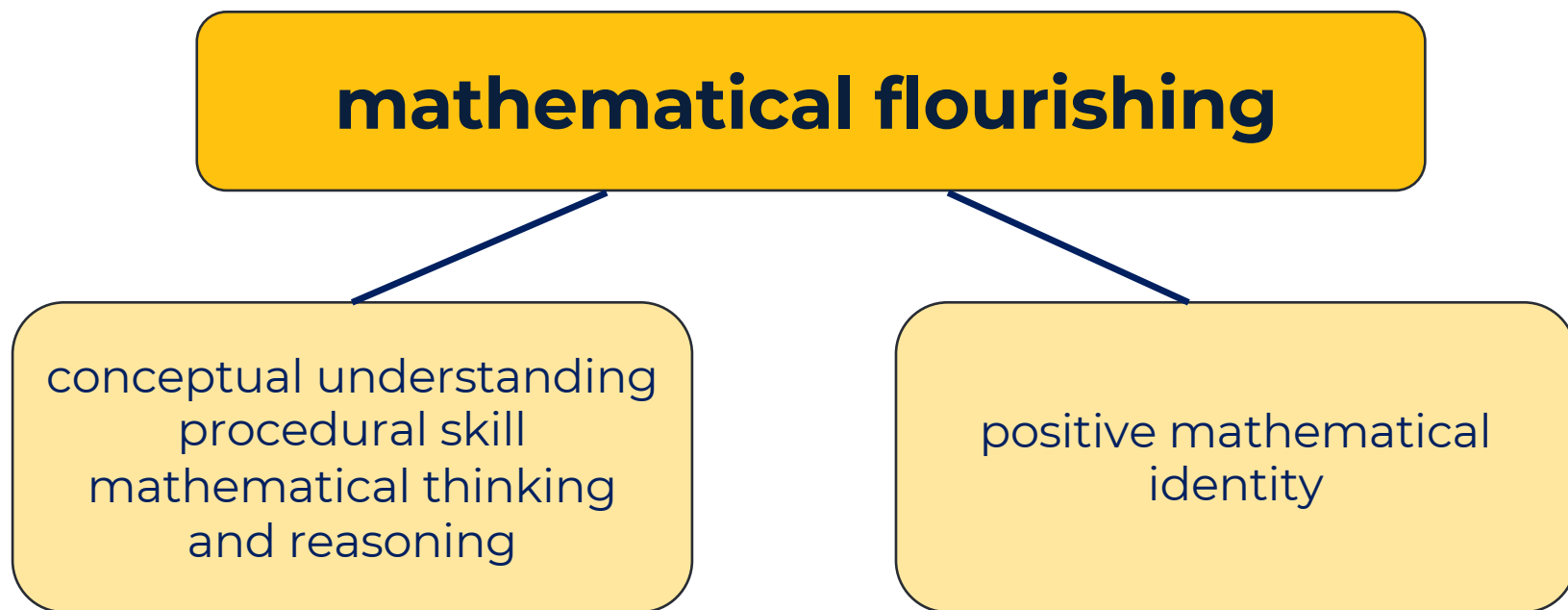
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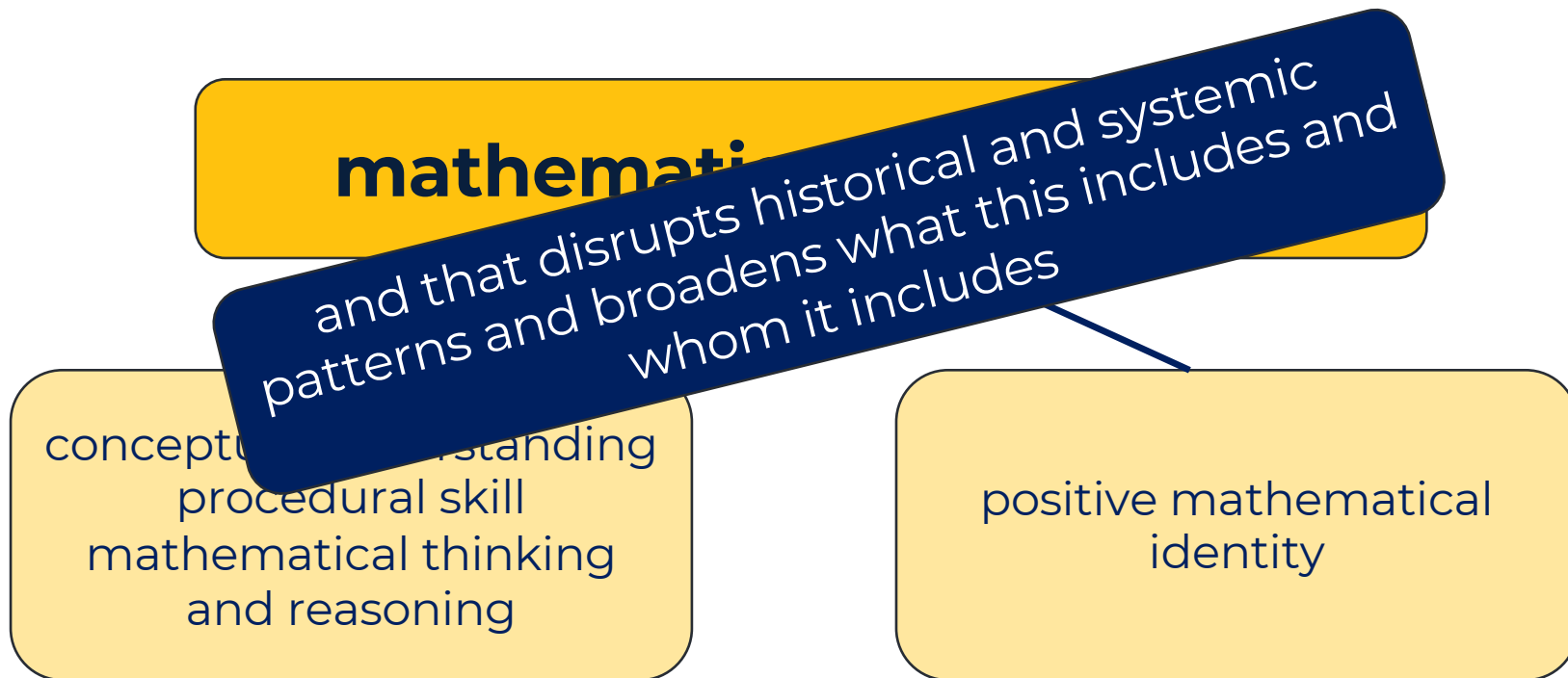
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mathematical flourishing







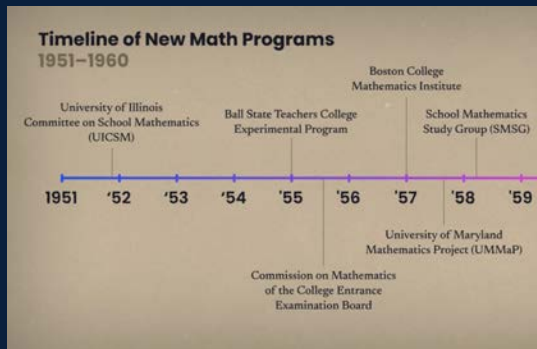
These are not new goals.

We have been here before, with
many of the same concerns.

Many times.



A 1953 issue of *The Mathematics Teacher* stated that “competence in mathematics widespread among our people is essential for the preservation of our society”.



1950-60s

Schools' Back-to-Basics Drive Found to Be Working in Math

By EDWARD B. FISKE

The back-to-basics movement in education that began in the 1970's has succeeded in assuring that almost every American high school graduate can handle simple mathematics, new testing data released yesterday suggest. But the data show that virtually no progress has been made on developing more complicated mathematical skills, normally taught in high school, that are increasingly sought by employers.

Only half the nation's 17-year-olds, for example, can solve mathematics problems at the junior high school level and fewer than one in 15 can cope with problems at the high school level that take several steps or involve algebra or geometry.

Students Up From Bottom

"Thanks to the back-to-basics thrust, we've brought up the students who were at the bottom," said Gregory Arrig, president of the Educational Testing Service of Lawrence Township, N.J. "Now the challenge is to do something about the upper end of the academic scale."

The new data were the latest mathematics survey from "The Nation's Report Card," or the National Assessment of Educational Progress. The results confirmed trends apparent in a study of students' reading capabilities released earlier this year.

The report, "The Mathematics Report Card: Are We Measuring Up," included these findings:

- Most of the gains in basic skills were a result of improvements among black and Hispanic students and among those living in the Southeast.
- Teachers have recently begun to assign more homework and do more testing of students in the classroom.
- Most mathematics teaching is rather dull, with few teachers using calculators, computers and other new technologies.
- Most students see mathemat-

'Moderately Complex'

Refer to the following graph. This graph shows how far a typical car travels after the brakes are applied.

A car is traveling 55 miles per hour. About how far will it travel after applying the brakes?

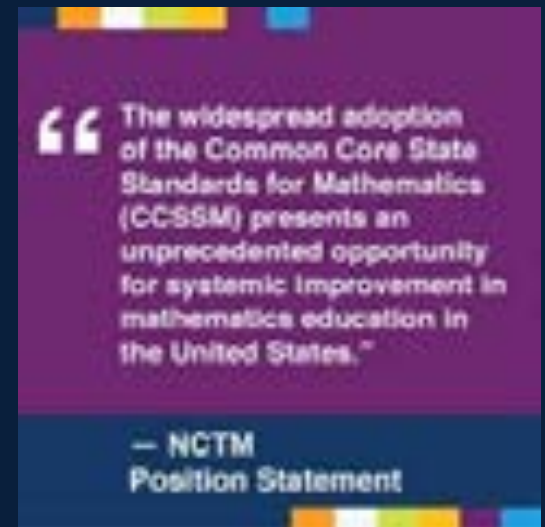
- 25 feet
- 200 feet
- 240 feet
- 350 feet
- I don't know

The New York Times/June 8, 1988
Question of the type found too difficult by half of 17-year-olds.
Continued on Page A25, Column 4

1970-1985



1990s



2010-

What have been the patterns?

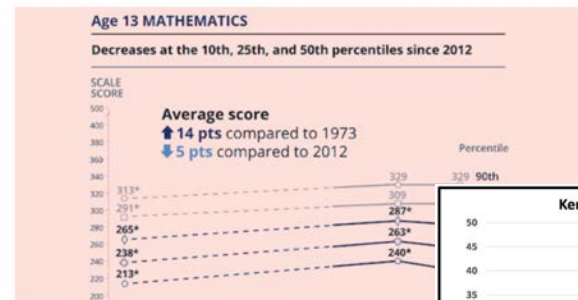
recurrent worries about achievement data and persistent narratives about “gaps”

U.S. STUDY SHOWS PUPIL ACHIEVEMENT AT LEVEL OF 1970

American elementary and secondary school pupils have made some educational progress in recent years, but they are only now reaching the achievement levels of students in 1970, according to a major Federal report issued today, which added that students are unprepared for the complex and demanding world of the 21st century. "Today's children seem to know about as much math and about as much science and read about as well as their parents did at that age about 20 years ago," Education Secretary Lamar Alexander said of the report, "Trends in Academic Progress."

October 1, 1991

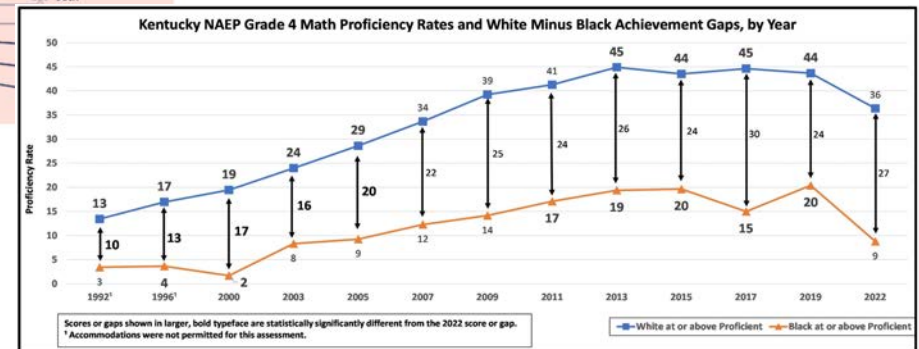
Long-Term NAEP Scores for 13-Year-Olds Drop for First Time Since Testing Began in 1970s — ‘A Matter for National Concern,’ Experts Say



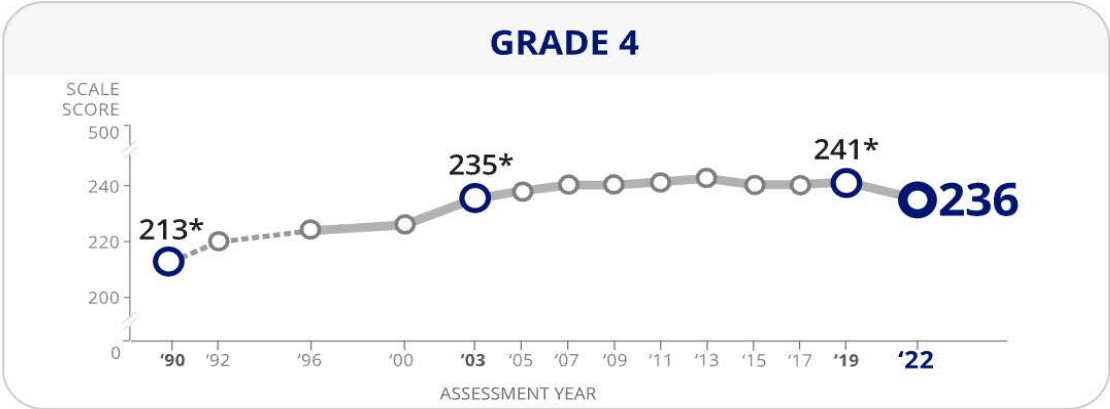
Two Decades of Progress, Nearly Gone: National Math, Reading Scores Hit Historic Lows



By Sarah D. Sparks — October 24, 2022 ⌚ 8 min read

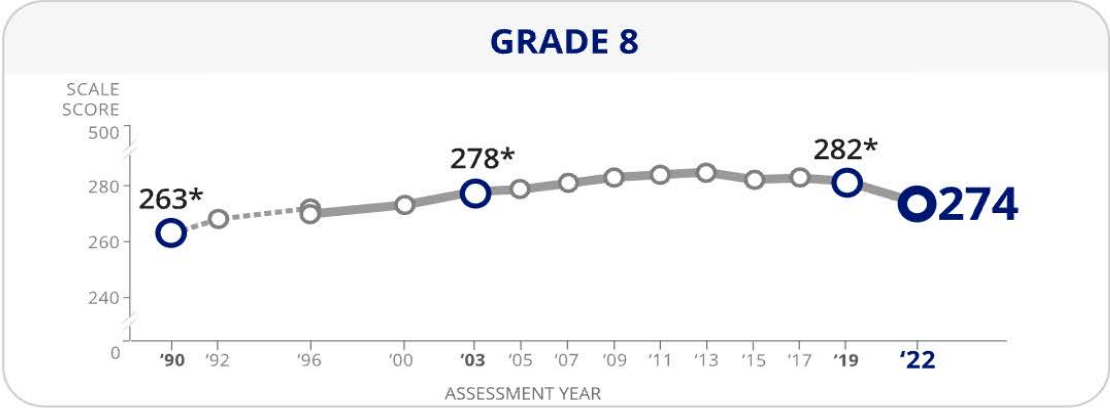


Yet, look at math score trends on NAEP:



23pts
 compared to 1990

5pts
 compared to 2019



12pts
 compared to 1990

8pts
 compared to 2019



Why Is the Nation Invested in Tearing Down Public Education?

We need a new education story with children and teachers at the center



By Deborah Loewenberg Ball — February 16, 2024



What have been other patterns?

repeated investments in designing and implementing new curriculum materials



But another pattern . . .

A limited understanding of the work of teaching and its relation to mathematics learning.



This is a fatal pattern.



Thus, after seven decades of cycles of “math reform,”
there is too little to show for it.

**Much (even most) mathematics classrooms look
much the same.**



Teachers explain how to “do” procedures.

Getting the answer right is the goal.

“Application” problems are contrived and seem irrelevant or meaningless to students.

Some students think they are “good at math” while others think they are bad at it.

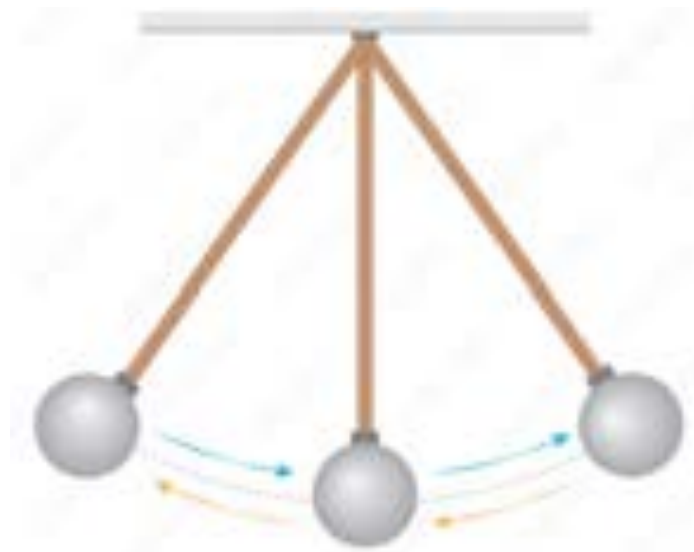
Black and Brown students and other systemically marginalized groups are pushed out.

Students are sorted by judgments about their mathematical “ability.”

Meanwhile, arguments and polarized discourses persist.



The pendulums swing back and forth.





In honor of Jim Lewis, an argument, complete with premises, definitions, claims, two theorems, a couple of lemmas, and evidence:

1. Fostering young people's mathematical flourishing is essential.
2. Many efforts have been made over decades to do this.
3. Still not a lot has changed inside mathematics classrooms, achievement is flat, and deep racial inequities persist.
4. **False dichotomies impede efforts to advance young people's mathematics flourishing.**
5. **A failure to disrupt these and to understand the work of teaching is a foundational flaw.**
6. Focusing on that work is a critical boundary-crossing agenda that depends on multiple forms of knowledge, humility, curiosity, respect, and nuance.



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We would have to confront a set of **false dichotomies**.

1. Direct instruction versus inquiry-based learning.
2. Teachers' mathematical knowledge versus teachers' relationships with students.
3. Focusing on getting right answers versus mathematical reasoning.
4. Instructional materials versus teacher autonomy and creativity.
5. Focusing on disrupting racism versus focusing on “the math.”
6. Individualized instruction versus collective classroom work.
7. Teacher education focused on social justice versus on practice.
8. “Struggling” learners versus “bright” students.



Definitions

di•chot•o•my: n. /dī'kädəmə/

(a) Division into two sharply defined or contrasting parts; division into two mutually exclusive categories or genera; binary classification. (b) A sharp or paradoxical contrast resulting from such a division; something paradoxical, ambivalent, or contradictory.

false: adj. /fôls/

(a) Of opinions, propositions, doctrines, representations: Contrary to what is true, erroneous.

false dichotomy: n. /fôls dī'kädəmə/

A logical fallacy in which a spectrum of possible options is misrepresented as an either-or choice between two mutually exclusive things.

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What makes any of these "a logical fallacy" in which a spectrum of possible options is misrepresented as an either-or choice between two mutually exclusive things"?

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TeachingWorks

What would it take
to disrupt these false
dichotomies and the
pendulum swings
they produce?

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TEACHINGWORKS

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Focus on the **work of
teaching** mathematics

26



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Creating a classroom environment for learning
Using and adapting curriculum materials
Assessing students' learning

Communicating with families

Focus on the **work of teaching** mathematics

Connecting mathematics to students' knowledge and contexts
Knowing and using mathematical knowledge

27



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What would it take to disrupt these false dichotomies and the pendulum swings they produce?

Creating a classroom environment for learning
Using and adapting curriculum materials
Assessing students' learning
Communicating with families

Focus on the **work of teaching** mathematics

Understand the density of the discretionary spaces inside teaching

Connecting mathematics to students' knowledge and contexts
Knowing and using mathematical knowledge



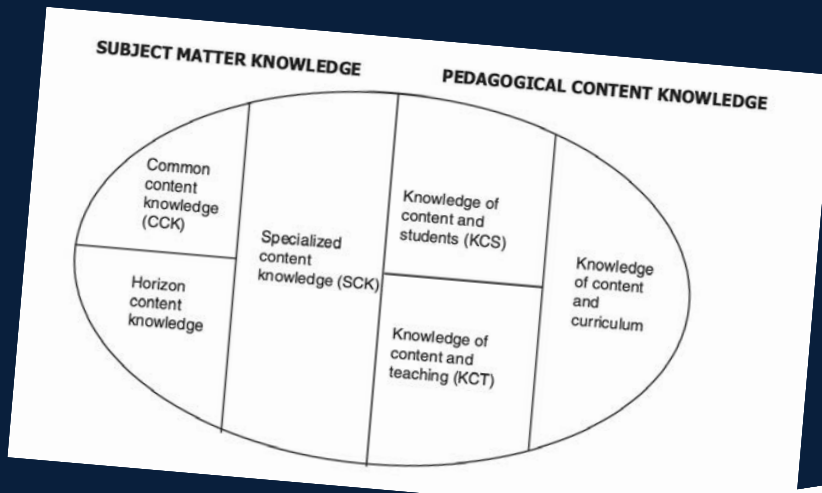
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Two theorems

1. Mathematical knowledge for teaching (MKT) is necessary but insufficient for improved mathematics teaching.
2. Good curriculum materials matter but materials do not by themselves teach.

The Unrealized Promise of High-Quality Instructional Materials
Overcoming barriers to faithful implementation requires changing teacher and leader mind-sets.
By David Stinner

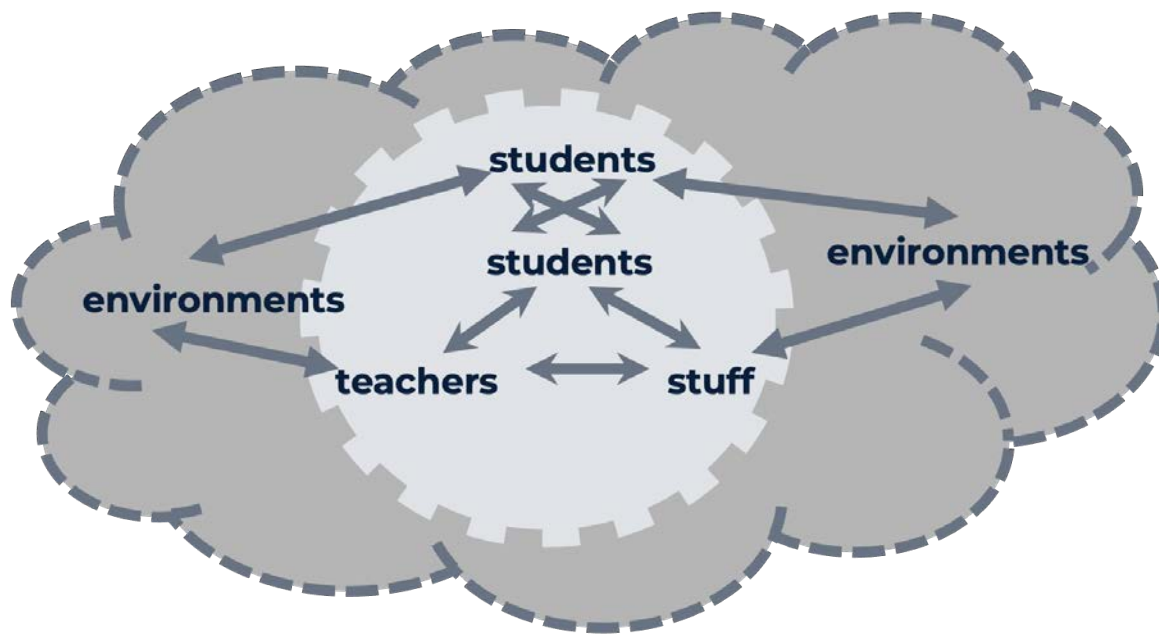
Two lemmas

1. The work of mathematics teaching denies and defies these dichotomies.
2. The work of mathematics teaching inherently entails tremendous discretion.

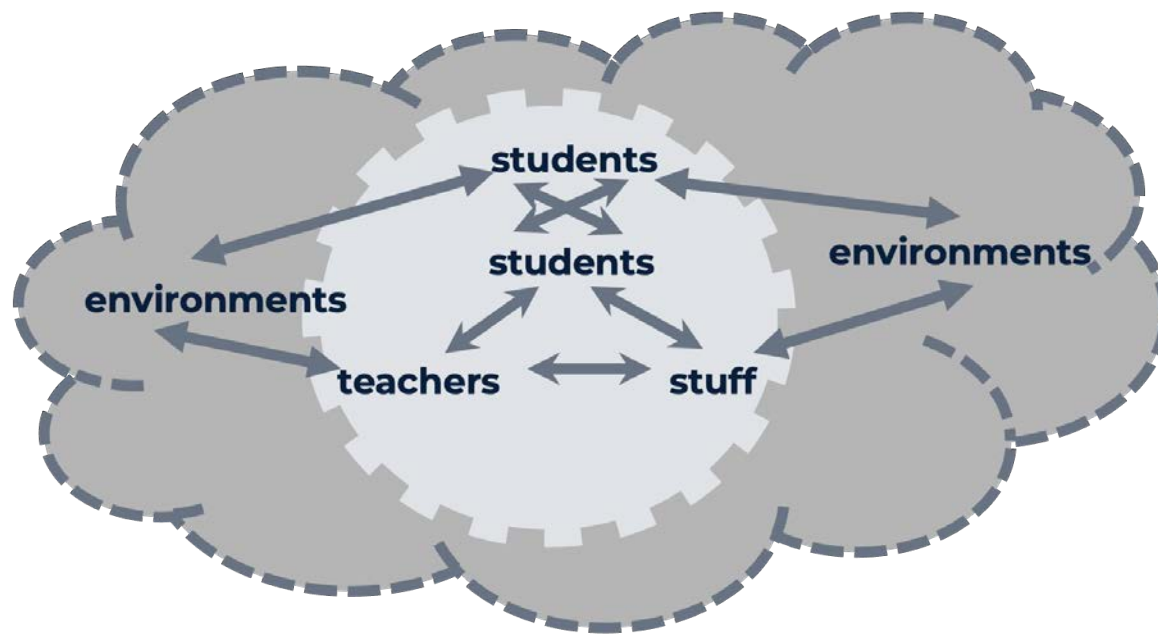
Creating a classroom materials
Using and adapting custom environment for learning
Assessing students' learning
Communicating with families
Focus on the **work of teaching** mathematics
Understand the density of the **discretionary spaces inside teaching**
Knowing and using mathematical knowledge in contexts

1. Direct instruction versus inquiry-based learning.
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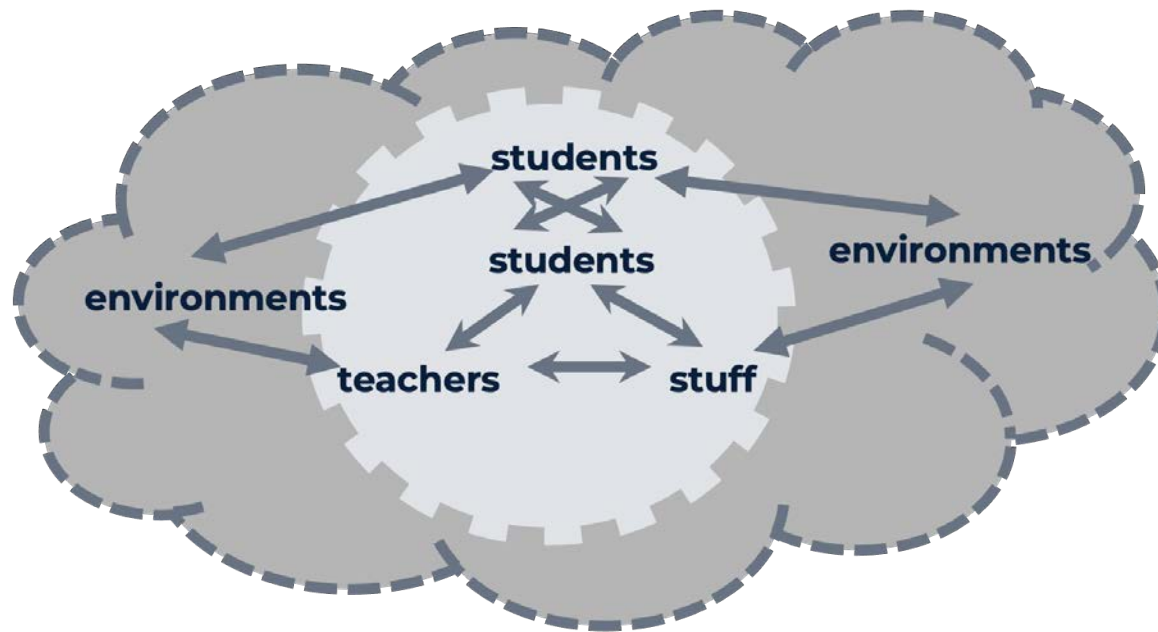




1. Teaching and learning are constructed interactively and interpretively in context.



1. Teaching and learning are constructed interactively and interpretively in context.
2. They take place within broad historical and socio-political environments.



1. **Teaching is powerful.** When it is done with care and judgment, students can thrive — learn mathematics, develop positive identities, learn to value others and work collectively.
2. Teaching also involves **enormous discretion.***
3. How that discretion is exercised can either reinforce racialized and oppressive patterns of social, personal, and epistemic injustice and harm, or it can **disrupt these patterns.**

(*Loewenberg Ball, 2018, in press; Lipsky, 1980)

Proof:

Advancing young people's mathematical flourishing requires centering the work of teaching mathematics, which depends on multiple forms of knowledge, humility, curiosity, respect, and nuance.



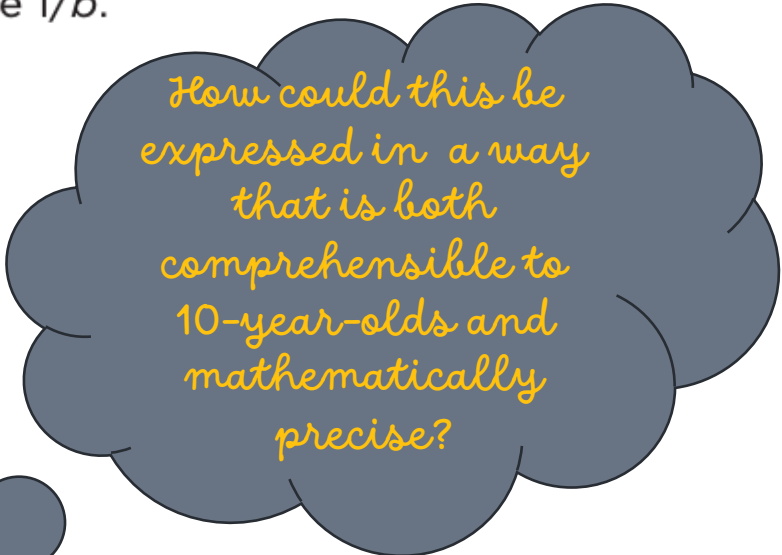
Fractions: The given instructional goal

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.



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How could this be expressed in a way that is both comprehensible to 10-year-olds and mathematically precise?

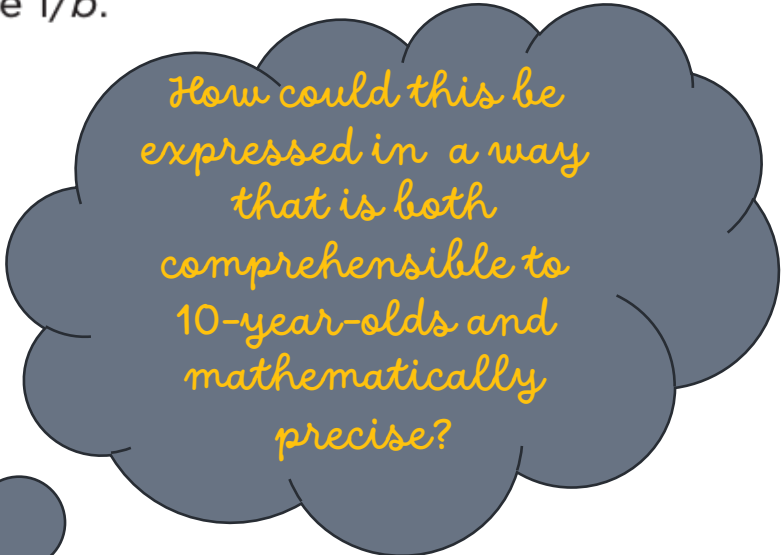
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NAMING A FRACTION

1. Figure out what the whole is.
2. Make sure that the whole is divided into equal parts. If not, make the parts equal.
3. Count how many equal parts there are. Call this number d .
4. Write $1/d$ to show one of the equal parts.
5. If more than 1 of those parts is shaded, count them (n) and write (n/d) .

NOTE: $d \neq 0$ and (in fourth grade) n must be a whole number.



How could this be expressed in a way that is both comprehensible to 10-year-olds and mathematically precise?

The given mathematics task from a “high-quality instructional material”

2. Juanita cut her string cheese into equal pieces as shown in the rectangles below. In the blanks below, name the fraction of the string cheese represented by the shaded part.



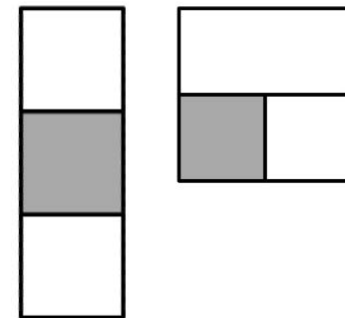
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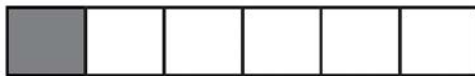
An adapted mathematics task

What fraction of each rectangle below is shaded gray?



The given mathematics task from a “high-quality instructional material”

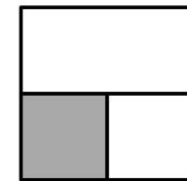
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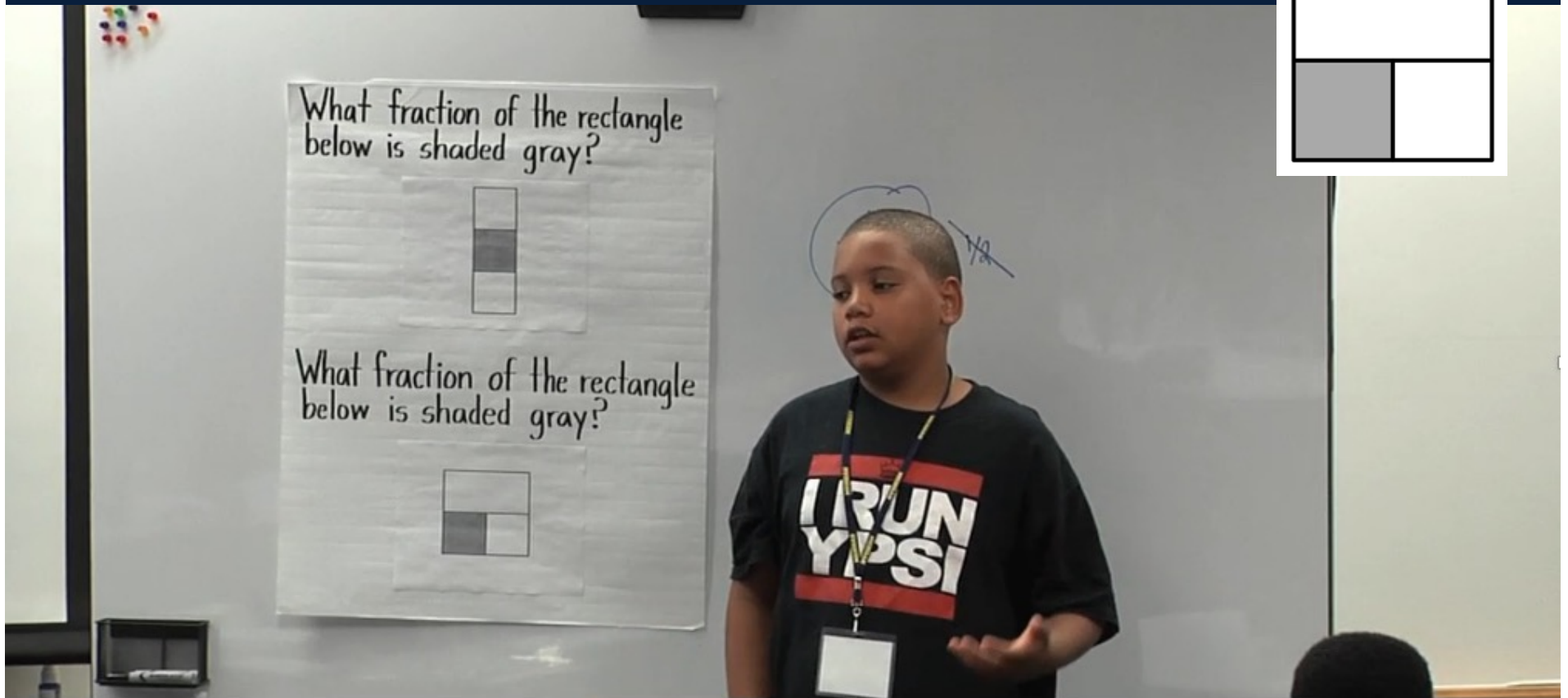
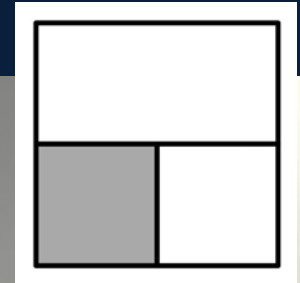
Both align with the learning goal. What is the same and what is different?

An adapted mathematics task

What fraction of each rectangle below is shaded gray?



Video: Antar, Gabriella, and Gabi



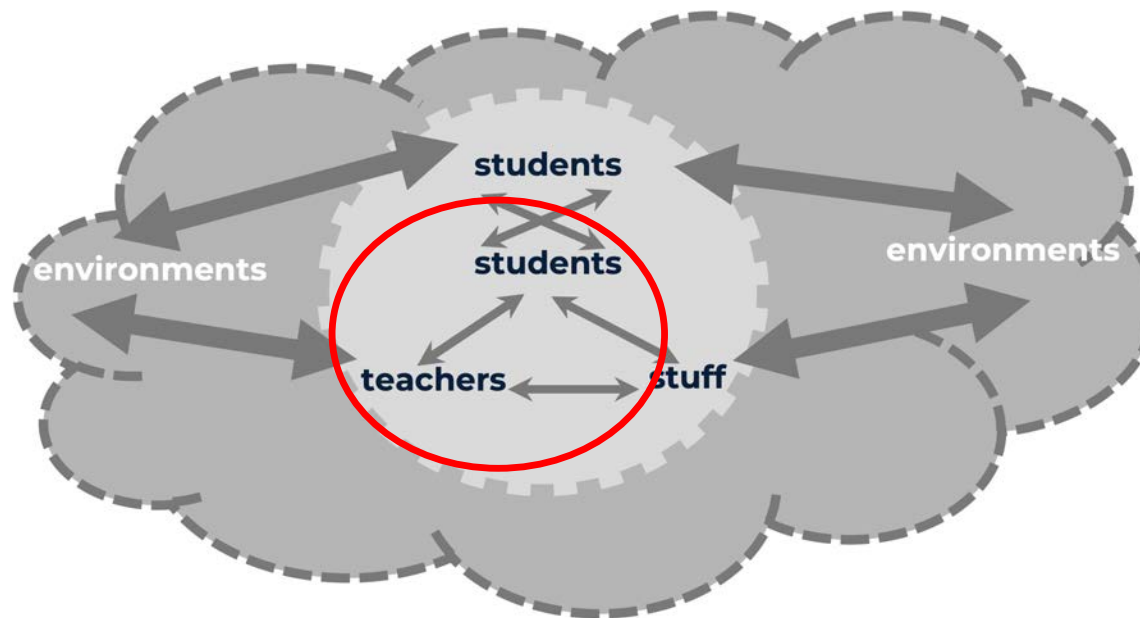
Antar: I think it's not a fraction because all the parts are not equally the same.



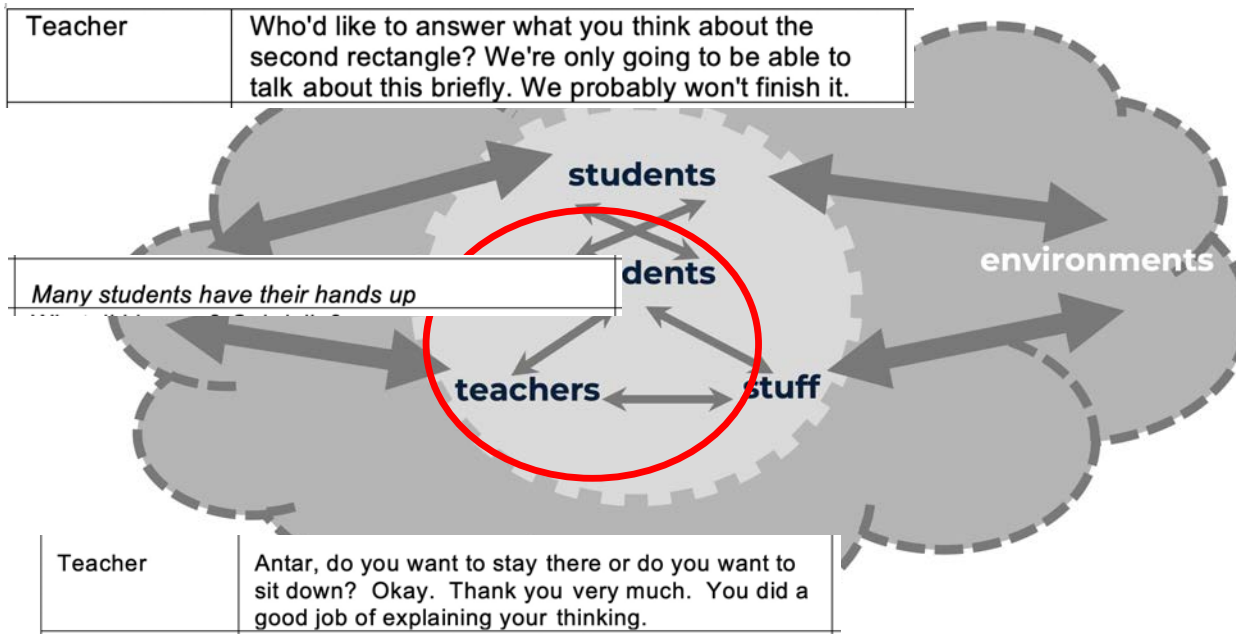
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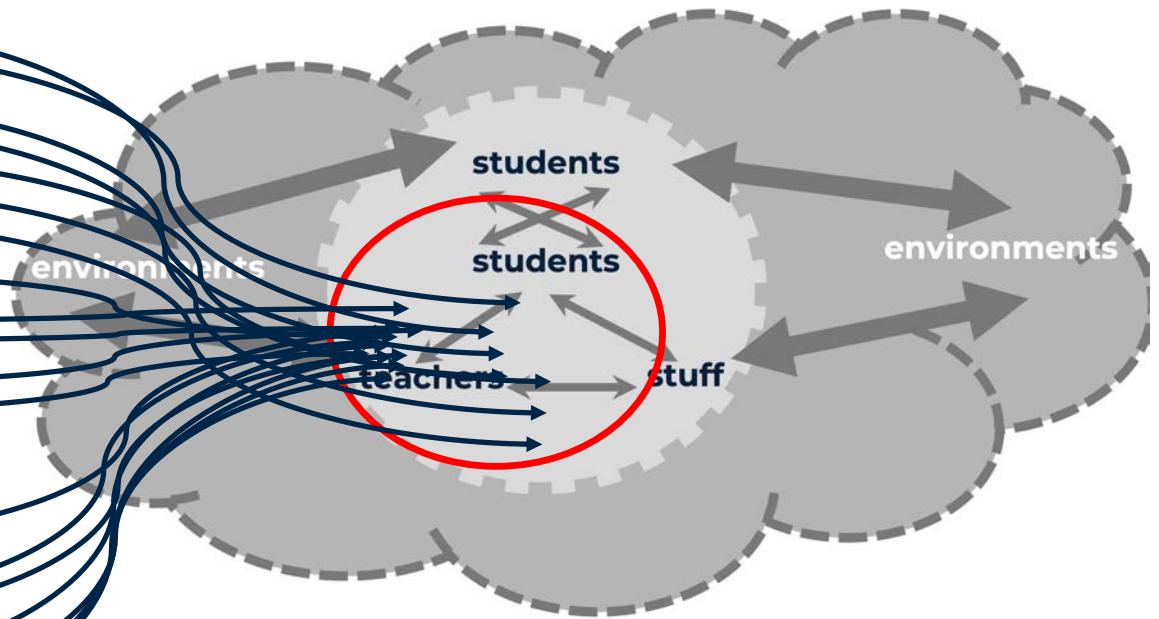
| | | |
|-----------|--|--|
| Teacher | Who'd like to answer what you think about the second rectangle? We're only going to be able to talk about this briefly. We probably won't finish it. | Launch discussion |
| Teacher | Who'd like to explain what you think? Antar, what do you think? | Choose student to call on |
| Teacher | Could you come up to the board and explain? Thank you. | Frame task for student who is presenting |
| Teacher | I really like the way that people who are coming to the board are doing today. You are explaining really well. | Acknowledge competence |
| Teacher | Here's a marker. Can you explain your thinking? | Provide material support |
| Antar | I think it's not a fraction because all of the parts are not equally the same shape. | Listen |
| Teacher | Can you say that one more time to the class? | Support presenter |
| Antar | I think it's not a fraction because all the parts are not equally the same. | Listen |
| Teacher | Can someone repeat what Antar said? Very nice, Antar. | Orient students to presenter |
| | <i>Many students have their hands up</i> | |
| Teacher | What did he say? Gabriella? | Choose student to call on |
| Gabriella | Oh. He said that he doesn't think it's a fraction because not all the parts are equal. | Listen |
| Teacher | Is that what you said? | Position first student as authority |
| Teacher | Okay, would someone like to comment on that? Agree or disagree with him? | Orient students to one another |
| Teacher | Okay, let's see, how about Gabi. | Choose student to call on |
| Gabi | I disagree. | Listen |
| Teacher | What do you think? | Pose question |
| Gabi | I think the fraction is one-fourth. | Listen |
| Teacher | One-fourth? Do you want to come up and say why you think it's one fourth? | Frame next step, support next presenter |
| Teacher | Antar, do you want to stay there or do you want to sit down? Okay. Thank you very much. You did a good job of explaining your thinking. | Position student with agency, acknowledge competence |
| Teacher | So, let's hear what Gabi's thinking. | Orient students to one another |
| Gabi | I think it's one-fourth because, like he said, all the fractions aren't the same, but you can make them the same by dividing a line down the middle. | Listen |
| Teacher | Here's something you can use so if someone wants to take it off again, they can. Okay, so now explain what you've done. Talk to the class, okay? | Provide material support |
| Gabi | I divided it down the middle because, since it's not equal, you have to make it equal. | Listen |
| Teacher | And so then you decided? | Probe |
| Gabi | It's one-fourth. | Listen |



| | | |
|-----------|--|--|
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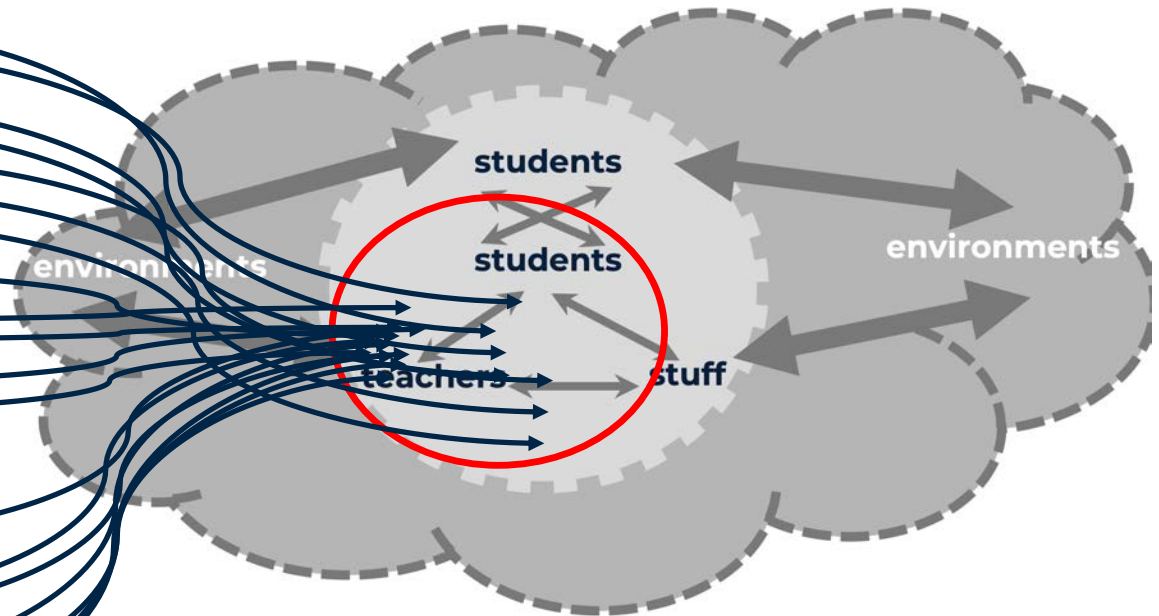


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| Teacher | Here's a marker. Can you explain your thinking? | Provide material support |
| Antar | I think it's not a fraction because all of the parts are not equally the same shape. | Listen |
| Teacher | Can you say that one more time to the class? | Support presenter |
| Antar | I think it's not a fraction because all the parts are not equally the same. | Listen |
| Teacher | Can someone repeat what Antar said? Very nice, Antar. | Orient students to presenter |
| | <i>Many students have their hands up</i> | |
| Teacher | What did he say? Gabriella? | Choose student to call on |
| Gabriella | Oh. He said that he doesn't think it's a fraction because not all the parts are equal. | Listen |
| Teacher | Is that what you said? | Position first student as authority |
| Teacher | Okay, would someone like to comment on that? Agree or disagree with him? | Orient students to one another |
| Teacher | Okay, let's see, how about Gabi. | Choose student to call on |
| Gabi | I disagree. | Listen |
| Teacher | What do you think? | Pose question |
| Gabi | I think the fraction is one-fourth. | Listen |
| Teacher | One-fourth? Do you want to come up and say why you think it's one fourth? | Frame next step, support next presenter |
| Teacher | Antar, do you want to stay there or do you want to sit down? Okay. Thank you very much. You did a good job of explaining your thinking. | Position student with agency, acknowledge competence |
| Teacher | So, let's hear what Gabi's thinking. | Orient students to one another |
| Gabi | I think it's one-fourth because, like he said, all the fractions aren't the same, but you can make them the same by dividing a line down the middle. | Listen |
| Teacher | Here's something you can use so if someone wants to take it off again, they can. Okay, so now explain what you've done. Talk to the class, okay? | Provide material support |
| Gabi | I divided it down the middle because, since it's not equal, you have to make it equal. | Listen |
| Teacher | And so then you decided? | Probe |
| Gabi | It's one-fourth. | Listen |



Teaching is dense with “discretionary spaces”

| | | |
|-----------|--|--|
| Teacher | Who'd like to answer what you think about the second rectangle? We're only going to be able to talk about this briefly. We probably won't finish it. | Launch discussion |
| Teacher | Who'd like to explain what you think? Antar, what do you think? | Choose student to call on |
| Teacher | Could you come up to the board and explain? Thank you. | Frame task for student who is presenting |
| Teacher | I really like the way that people who are coming to the board are doing today. You are explaining really well. | Acknowledge competence |
| Teacher | Here's a marker. Can you explain your thinking? | Provide material support |
| Antar | I think it's not a fraction because all of the parts are not equally the same shape. | Listen |
| Teacher | Can you say that one more time to the class? | Support presenter |
| Antar | I think it's not a fraction because all the parts are not equally the same. | Listen |
| Teacher | Can someone repeat what Antar said? Very nice, Antar. | Orient students to presenter |
| | <i>Many students have their hands up</i> | |
| Teacher | What did he say? Gabriella? | Choose student to call on |
| Gabriella | Oh. He said that he doesn't think it's a fraction because not all the parts are equal. | Listen |
| Teacher | Is that what you said? | Position first student as authority |
| Teacher | Okay, would someone like to comment on that? Agree or disagree with him? | Orient students to one another |
| Teacher | Okay, let's see, how about Gabi. | Choose student to call on |
| Gabi | I disagree. | Listen |
| Teacher | What do you think? | Pose question |
| Gabi | I think the fraction is one-fourth. | Listen |
| Teacher | One-fourth? Do you want to come up and say why you think it's one fourth? | Frame next step, support next presenter |
| Teacher | Antar, do you want to stay there or do you want to sit down? Okay. Thank you very much. You did a good job of explaining your thinking. | Position student with agency, acknowledge competence |
| Teacher | So, let's hear what Gabi's thinking. | Orient students to one another |
| Gabi | I think it's one-fourth because, like he said, all the fractions aren't the same, but you can make them the same by dividing a line down the middle. | Listen |
| Teacher | Here's something you can use so if someone wants to take it off again, they can. Okay, so now explain what you've done. Talk to the class, okay? | Provide material support |
| Gabi | I divided it down the middle because, since it's not equal, you have to make it equal. | Listen |
| Teacher | And so then you decided? | Probe |
| Gabi | It's one-fourth. | Listen |



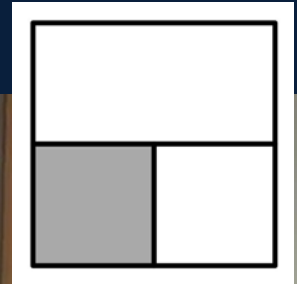
25 in 2:21

(Some of) the teaching and learning goals:

- Helping students understand fractions
- Supporting collective mathematical work
- Supporting mathematical listening, appraising validity, questioning
- Disrupting patterns of what is seen as mathematical competence and who is seen as mathematically competent
- Supporting the development of positive mathematical identities



Video: Antar, Gabriella, Gabi, Virshawn, Marquis, and Kassie



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What would
commonly
happen
next?

What are the
possible
results?



In the next moment, what is likely to happen?

NORMALIZED NEXT MOVES

RESULTS

In the next moment, what is likely to happen?

NORMALIZED NEXT MOVES

- “Kassie, I see what you are saying, but remember that Gabi just showed us what the whole is and that the gray square is one-fourth.”

RESULTS

- Kassie’s answer is signaled to be incorrect and she is positioned as not having contributed to the work.

In the next moment, what is likely to happen?

NORMALIZED NEXT MOVES

- “Kassie, I see what you are saying, but remember that Gabi just showed us what the whole is and that the gray square is one-fourth.”
- “Kassie, it is really important to identify what the whole is first. Who can show Kassie what the whole is here?”

RESULTS

- Kassie’s answer is signaled to be incorrect and she is positioned as not having contributed to the work.
- Kassie is called out for not knowing what the whole is and she is sidelined.

In the next moment, what is likely to happen?

NORMALIZED NEXT MOVES

- “Kassie, I see what you are saying, but remember that Gabi just showed us what the whole is and that the gray square is one-fourth.”
- “Kassie, it is really important to identify what the whole is first. Who can show Kassie what the whole is here?”
- “Thumbs up if you agree with Kassie; thumbs down if you disagree.”

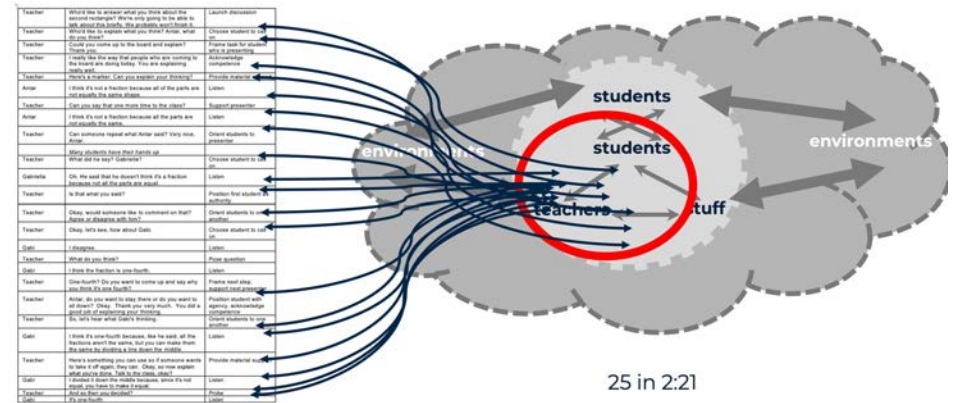
RESULTS

- Kassie’s answer is signaled to be incorrect and she is positioned as not having contributed to the work.
- Kassie is called out for not knowing what the whole is and she is sidelined.
- Kassie’s solution is “voted” on by her classmates.


- A discretionary space is where interpretations, next moves, comments, or questions are necessarily determined by the teacher—and not by a policy or curriculum.
- These interpretations and actions are learned through firsthand experience in society and in school.
- These interpretations and actions are also habituated.

(Loewenberg Ball, 2018; Ngo, 2017, Noel, 2018, Sfard)

Discretionary spaces are inherent in teaching



The density of discretionary spaces related to advancing children's mathematical flourishing

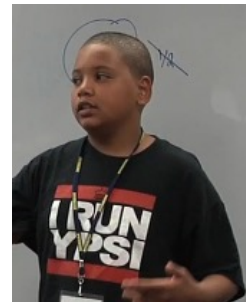


What are some of the discretionary spaces and risks?

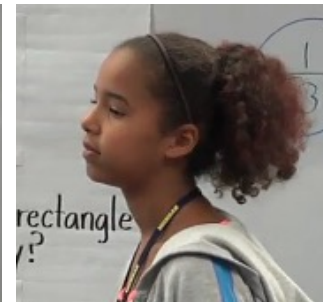


The density of discretionary spaces related to advancing children's mathematical flourishing

What are some of the discretionary spaces and risks?



Antar



Gabi



Kassie

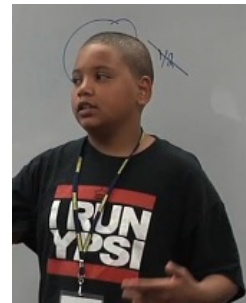


The density of discretionary spaces related to advancing children's mathematical flourishing

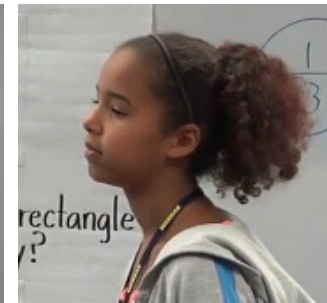
- How are these three different Black children—Antar, Gabi, and Kassie—positioned in front of their classmates—as contributing to the mathematics, as lacking understanding? Are their brilliance and humanity seen?
- What is signaled about being a “doer of mathematics”—both what and who?
- What mathematical understanding is developing in the class?
- How are Antar, Gabi, and Kassie experiencing their teacher, their peers, this lesson?
- What are the other children in the class learning about Black children, about who and what it means to be “smart”?

(Gholson, 2021; Wilkes, 2021; Gholson & Martin, 2014; Langer-Osuna, 2015, 2017; Leonard & Martin, 2013; Martin, 2012, 2015)

What are some of the discretionary spaces and risks?



Antar



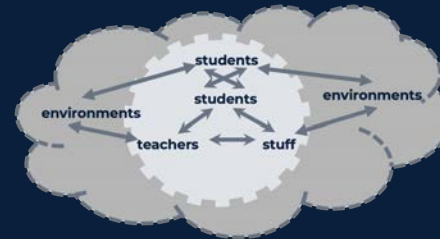
Gabi



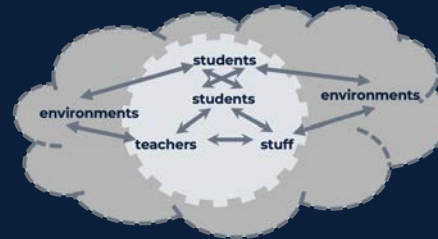
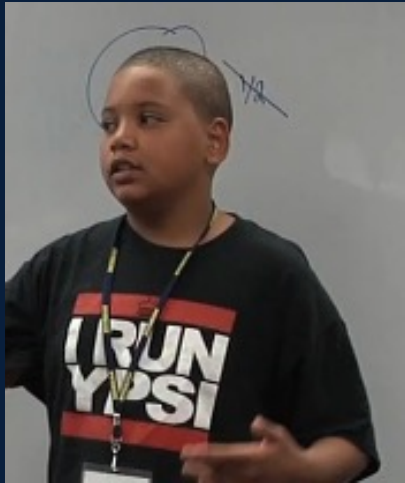
Kassie



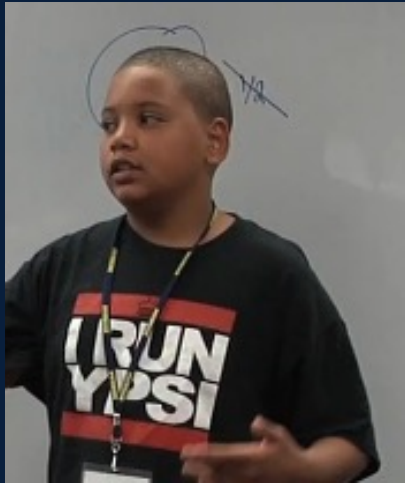
In this case, what is some of the work of teaching mathematics for students' mathematical flourishing?



In this case, what is some of the work of teaching mathematics for students' mathematical flourishing?

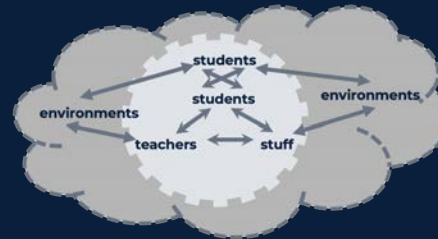


In this case, what is some of the work of teaching mathematics for students' mathematical flourishing?

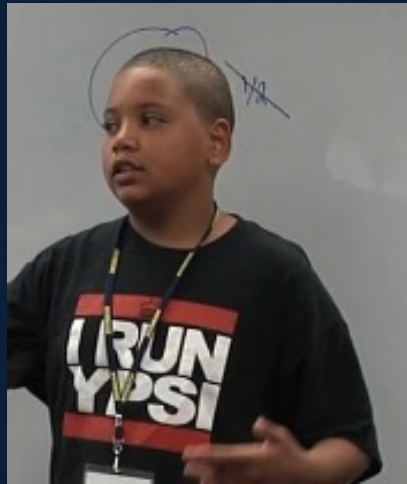


Seeing Antar's explanation and Kassie's question as key to the class's work

Knowing and using mathematics in teaching (MKT)



In this case, what is some of the work of teaching mathematics for students' mathematical flourishing?

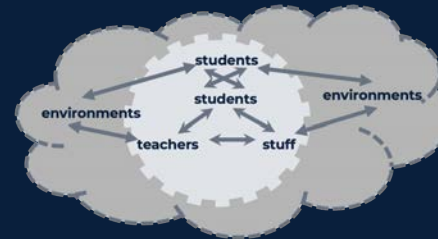


Seeing Antar's explanation and Kassie's question as key to the class's work

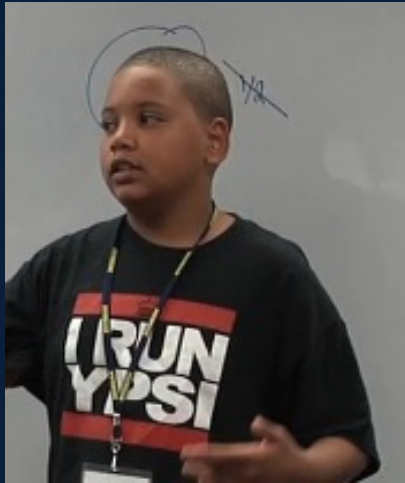
Seeing what Marquis is doing instead of what one wants to hear

Knowing and using mathematics in teaching (MKT)

Taking as axiomatic the brilliance of Black children



In this case, what is some of the work of teaching mathematics for students' mathematical flourishing?



Gholson & Martin, 2014; Langer-Osuna, 2015, 2017; Leonard & Martin, 2013; Martin, 2012, 2015

Seeing Antar's explanation and Kassie's question as key to the class's work

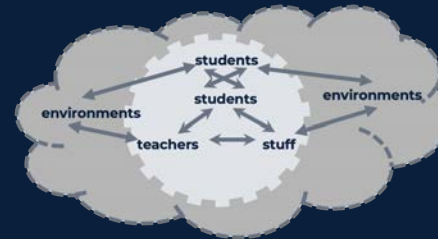
Seeing what Marquis is doing instead of what one wants to hear

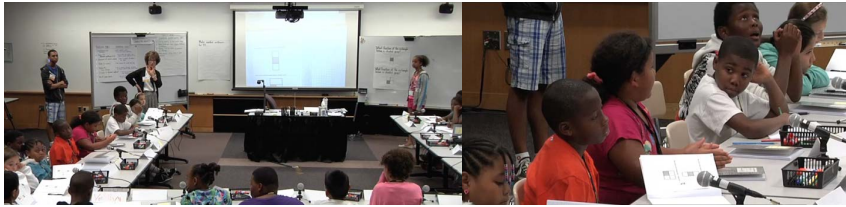
Having things to DO that are not the norm

Knowing and using mathematics in teaching (MKT)

Taking as axiomatic the brilliance of Black children

Having a repertoire of practices that can be adapted and used in contexts

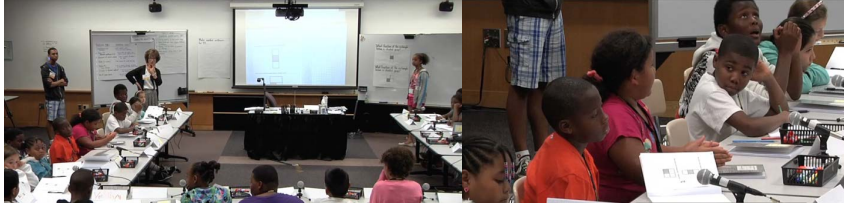




Identifying mathematical learning goals

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.



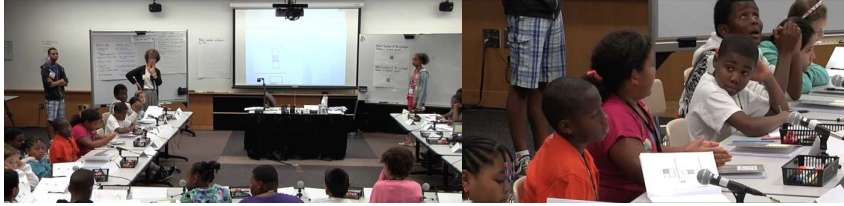


Teaching and learning mathematics

Identifying mathematical learning goals

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.





Teaching and learning mathematics

- Understanding mathematics oriented to others' thinking
- Helping others understand fractions
- Supporting collective mathematical work
- Listening to others' ideas
- Seeing and hearing students
- Supporting mathematical listening, appraising validity, questioning
- Disrupting patterns of what is seen as mathematical competence and who is seen as mathematically competent
- Supporting the development of positive mathematical identities

Identifying mathematical learning goals

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.



Review: The argument of today's talk

1. Fostering young people's mathematical flourishing is essential.
2. Many efforts have been made over decades to do this.
3. Still not a lot has changed inside mathematics classrooms, achievement is flat, and deep racial inequities persist.
4. False dichotomies impede efforts to advance young people's mathematics flourishing.
5. A failure to disrupt these and to understand the work of teaching is a foundational flaw.
6. Focusing on that work is a critical boundary-crossing agenda that depends on multiple forms of knowledge, humility, curiosity, respect, and nuance.



M MARSAL EDUCATION

TeachingWorks

What would it take
to disrupt false
dichotomies and the
pendulum swings
they produce?

UNIVERSITY OF MICHIGAN
MARSAL FAMILY SCHOOL OF EDUCATION
TEACHINGWORKS

Creating a classroom materials .
Using and adapting curriculum materials .
Assessing students' learning
Communicating with families

Focus on the **work of teaching** mathematics.

Connecting mathematical knowledge
Knowing and using mathematics to students' contexts

Understand and leverage the density of
the discretionary spaces inside teaching

69



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Thank you, Jim!



2024

Elementary Mathematics Laboratory

June 17–21

The School at Marygrove
Detroit, MI

June 24–28

Grand Rapids Public Schools
Grand Rapids, MI



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THANK YOU!



dball@umich.edu



<https://deborahloewenbergball.com/>



@deborah_ball



Credits

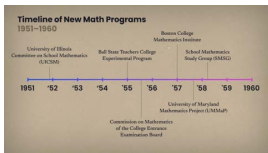


Image on slide 10:

A timeline of new math reform organisations in the 1950s, from “What Happened to ‘New Math’?” by Asher Isbrucker, *Medium*, April 20, 2021.

Retrieved from <https://medium.com/age-of-awareness/what-happened-to-new-math-eeb8522fc695>

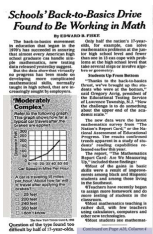


Image on slide 10:

Screenshot of “School’s Back-to-Basics Drive Found to be Working in Math,” by Edward D. Fiske, *New York Times*, June 8, 1988.

Retrieved from <https://www.nytimes.com/1988/06/08/us/schools-back-to-basics-drive-found-to-be-working-in-math.html>



Image on slide 10:

Cover of *Curriculum and Evaluation Standards for School Mathematics*, by the National Council of Teachers of Mathematics Commission on Standards for School Mathematics, 1989.

Retrieved from <https://archive.org/details/curriculumevalua00nati/mode/2up>

Credits

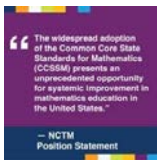


Image on slide 10:
Graphic from “Supporting the Common Core State Standards for Mathematics,” by the National Council of Teachers of Mathematics, August 2013.
Retrieved from <https://www.nctm.org/ccssmposition/>

U.S. STUDY SHOWS PUPIL ACHIEVEMENT AT LEVEL OF 1970

American elementary and secondary school pupils have made some educational progress in recent years, but they are only now reaching the achievement levels of students in 1970, according to a major federal report issued today, which added that students are unprepared for the complex and demanding world of the 21st century. “Today’s children seem to know about as much math and about as much science and read about as well as their parents did at that age about 50 years ago,” Education Secretary Lamar Alexander said of his report, “Trends in Academic Progress.”

Image on slide 11:
Screenshot from “U.S. study shows pupil achievement at level of 1970,” by Karen De Witt, *New York Times*, October 1, 1991.
Retrieved from <https://www.nytimes.com/1991/10/01/us/us-study-shows-pupil-achievement-at-level-of-1970.html>

Long-Term NAEP Scores for 13-Year-Olds Drop for First Time Since Testing Began in 1970s — ‘A Matter for National Concern,’ Experts Say

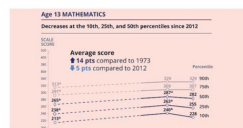


Image on slide 11:
Screenshot from “Long-Term NAEP Scores for 13-Year-Olds Drop for First Time Since Testing Began in 1970s — ‘A Matter for National Concern,’ Experts Say,” by Kevin Mahnken, *The74*, October 14, 2021.
Retrieved from <https://www.the74million.org/article/naep-long-term-unprecedented-performance-drop-american-13-year-olds/>

Credits

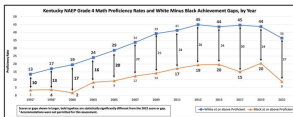


Image on slide 11:

Graphic from “What happened to Kentucky’s NAEP achievement gaps for math?,” by Richard Innes, Bluegrass Institute, October 30, 2022.

Retrieved from <https://bipps.org/blog/what-happened-to-kentuckys-naep-achievement-gaps-for-math>

Two Decades of Progress, Nearly Gone: National Math, Reading Scores Hit Historic Lows

By Sarah D. Sparks — October 24, 2022 8 min read

Image on slide 11:

Headline from “Two decades of progress, nearly gone; National math, reading scores hit historic lows,” by Sarah D. Sparks, *Education Week*, October 24, 2022.

Retrieved from <https://www.edweek.org/leadership/two-decades-of-progress-nearly-gone-national-math-reading-scores-hit-historic-lows/2022/10>

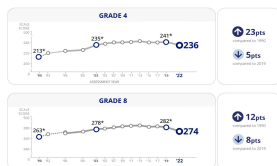


Image on slide 12:

Figure from 2022 NAEP mathematics assessment highlights

Retrieved from <https://www.nationsreportcard.gov/highlights/mathematics/2022/>

Credits

Why Is the Nation Invested in Tearing Down Public Education?

We need a new education story with children and teachers at the center

By Deborah Loewenberg Ball — February 16, 2024



Images on slide 13:

Graphics from “Why Is the Nation Invested in Tearing Down Public Education?,” by the Deborah Loewenberg Ball, *Education Week*, February 16, 2024.

Retrieved from <https://www.edweek.org/teaching-learning/opinion-why-is-the-nation-invested-in-tearing-down-public-education/2024/02>



Image on slide 14:

Cover of *Linear Algebra*, 1965.

Retrieved from <https://www.amazon.com/Linear-Algebra-Fundamental-Mathematical-Structures/dp/B002NB51BM>

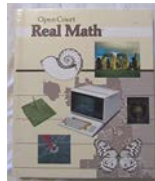


Image on slide 14:

Cover of *Open Court Real Math, Grade 7*, 1991.

Retrieved from <https://www.amazon.com/Open-Court-Real-Math-Grade/dp/081260637X>

Credits



Image on slide 14:

Cover of *Connected Mathematics Variables and Patterns*, 2003.

Retrieved from <https://www.amazon.com/CONNECTED-MATHEMATICS-VARIABLES-PATTERNS-STUDENT/dp/0131808168>

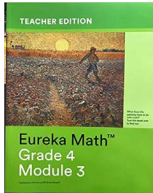


Image on slide 14:

Cover of *Eureka Math, A Story of Units: Grade 4, Module 3*, 2015.

Retrieved from <https://www.amazon.com/Eureka-Math-Grade-Module-Teachers/dp/1632553724>

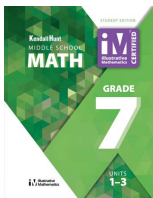


Image on slide 14:

Cover of *Illustrative Mathematics: Grade 7*, 2019.

Retrieved from <https://k12.kendallhunt.com/product/illustrative-mathematics-grade-7-student-edition-set>

Credits



Image on slide 15:

Photo from “What do teachers need this school year? Laura McClure from TED-Ed resolved to find out,” by Laura McClure, TED, September 7, 2016.

Retrieved from <https://ideas.ted.com/how-to-help-a-teacher-out/>



Image on slide 18:

“Two groups of people arguing and fighting.”

Retrieved from https://www.freepik.com/premium-vector/two-groups-people-arguing-fighting-conflict-among-people-angry-characters-having-argument-disagreement-vector-illustration-colleagues-having-debate-misunderstanding_21715825.htm

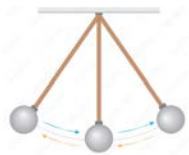


Image on slide 19:

“Pendulum, energy. Three forces work directly on the pendulum. Conservation of energy.”

Retrieved from <https://stock.adobe.com/images/pendulum-energy-three-forces-work-directly-on-the-pendulum-conservation-of-energy/413009447>

Credits

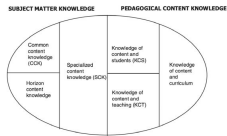


Image on slide 30:
 Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407.
<https://doi.org/10.1177/0022487108324554>

The Unrealized Promise of High-Quality Instructional Materials

Overcoming barriers to faithful implementation requires changing teacher and leader mind-sets.

By David Steiner

Image on slide 30:
 Headline from “The Unrealized Promise of High-Quality Instructional Materials” by David Steiner, *State Education Standard*.
 Retrieved from <https://www.nasbe.org/the-unrealized-promise-of-high-quality-instructional-materials/>

2. Shade in the string cheese with equal pieces as shown in the rectangles below. In the blanks below, name the fraction of the string cheese represented by the shaded part.

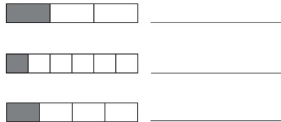


Image on slides 40–42:
 Problem from Eureka Math, Grade 3, Module 5, Lesson 1
 Retrieved from https://cdn2.hubspot.net/hubfs/3454910/Florida%20adoption%20materials/EurekaMath_G3_M5_UTE_FL.pdf