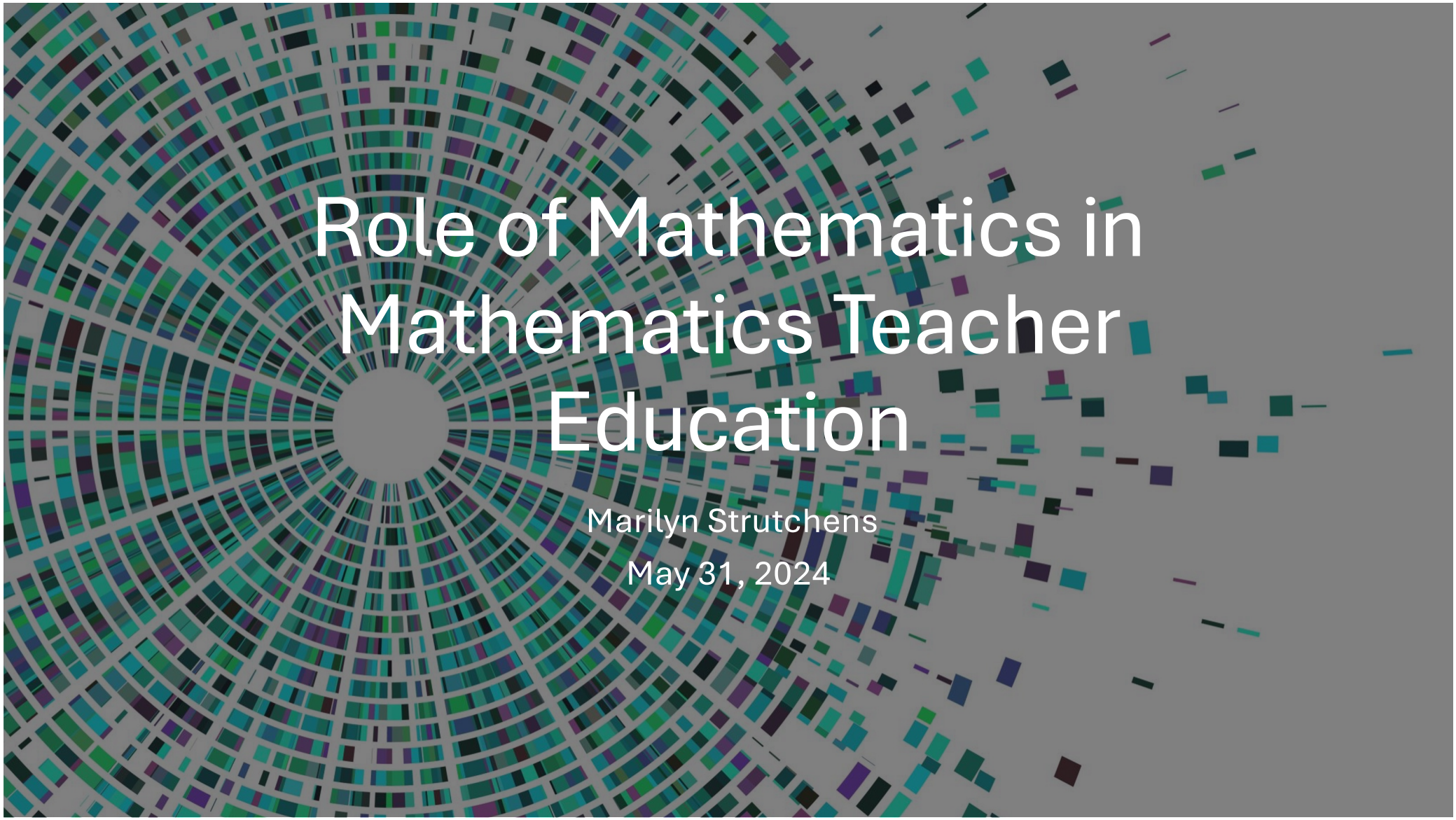


Role of Mathematics in Mathematics Teacher Education

Marilyn Strutchens

May 31, 2024

In the 21st century, school mathematics ought to connect the formal, abstract, generalized mathematics of the academy to the social and cultural backgrounds of the learners. Our ancestors developed the mathematics of Plato and Euclid, and they developed the mathematics of the rainforest. Both traditions support the multifaceted structure that underlies and drives modern society. Mathematics is both a finished product and a work in progress. It has a polished deductive aspect and simultaneously a human face. School mathematics needs to honor that complexity.

(Jeremy Kilpatrick, 2022, p. 6)

This statement aligns with the following recommendation from the National Council of Teachers of Mathematics' *Catalyzing Change* Series which states:

"*Each and every student should learn the Essential Concepts of mathematics in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics."*

Essential Concepts in High School Mathematics

Essential Concepts in Number

Essential Concepts in Algebra and Functions

- **Focus 1:** Algebra
- **Focus 2:** Connecting Algebra to Functions
- **Focus 3:** Functions

Essential Concepts in Statistics and Probability

- **Focus 1:** Quantitative Literacy
- **Focus 2:** Visualizing and Summarizing Data
- **Focus 3:** Statistical Inference
- **Focus 4:** Probability

Essential Concepts in Geometry and Measurement

- **Focus 1:** Measurement
- **Focus 2:** Transformations
- **Focus 3:** Geometric Arguments, Reasoning, and Proof
- **Focus 4:** Solving Applied Problems and Modeling in Geometry

Common Vision Project

The Common Vision project brought together leaders from five professional associations in the mathematical sciences –

- American Mathematical Association of Two-Year Colleges (AMATYC),
- American Mathematical Society (AMS),
- American Statistical Association (ASA),
- Mathematical Association of America (MAA), and
- Society for Industrial and Applied Mathematics (SIAM)

to collectively consider undergraduate mathematics curricula and ways to improve education in the mathematical sciences. (http://www.maa.org/programs/faculty-and-departments/common-vision)

(Saxe & Braddy, 2015)

Common Vision Report's Recommendations

- The status quo is unacceptable.
- They called on the community to
  - Update curricula,
  - Articulate clear pathways between curricula driven by changes at the K–12 level and the first courses students take in college,
  - Scale up the use of evidence-based pedagogical methods,
  - Find ways to remove barriers facing students at critical transition points (e.g., placement, transfer) and
  - Establish stronger connections with other disciplines.
- Asked institutions to provide faculty with training, resources, and rewards for their efforts to adapt curricula, develop new courses, and incorporate pedagogical tools and technology to enhance student learning.

MAA

Instructional Practices Guide

- Is designed as a “how to” guide focused on mathematics instruction at the undergraduate level.
- It is based on the concept that effective teaching is supported by three foundational types of practices: classroom practices, assessment practice, and course design practices all informed by empirical research as well as the literature on technology and equity.

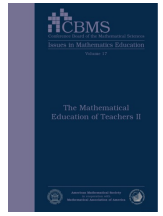
(The Mathematical Association of America, Inc., 2018)

Transforming Post-Secondary Education in Mathematics (TPSE Math)

- Sponsored by Carnegie Corporation of New York, the Alfred P. Sloan Foundation, and the National Science Foundation, aims to effect constructive change in mathematics education at U.S. community colleges, 4-year colleges, and research universities.
- ***Vision***: Post-secondary education in mathematics will enable any student, regardless of his or her chosen program of study, to develop the mathematical knowledge and skills necessary for productive engagement in society and in the workplace.
- ***Mission***: TPSE Math will facilitate an inclusive movement to strengthen post-secondary education in mathematics by working closely with--and mobilizing when necessary--faculty leaders, university administrations, membership associations, and relevant disciplinary societies in the pursuit of mathematically rich and relevant education for all students, whatever their chosen field of study. TPSE Math will identify innovative practices where they exist, advocate for innovation where they do not, and work with and through partners to implement and scale effective practices.

The Mathematical Education of Teachers II

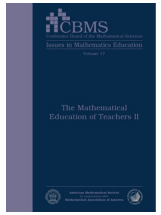
- The report makes recommendations for the mathematics that teachers should know and how they should come to know that mathematics. It urges greater involvement of mathematicians and statisticians in teacher education so that the nation's mathematics teachers have the knowledge, skills, and dispositions needed to provide students with a mathematics education that ensures high school graduates are college- and career-ready as envisioned by the Common Core State Standards.
- At the same time, MET II reiterates and elaborates themes of the first MET report:
  - There is intellectual substance in school mathematics.
  - Proficiency with school mathematics is necessary but not sufficient mathematical knowledge for a teacher.
  - The mathematical knowledge needed for teaching differs from that of other professions.
  - Mathematical knowledge for teaching can and should grow throughout a teacher's career. (Conference Board of the Mathematical Sciences, 2012)



The Mathematical Education of Teachers II

- **Recommendation 1.** Prospective teachers need mathematics courses that develop a solid understanding of the mathematics they will teach.
- **Recommendation 2.** Coursework that allows time to engage in reasoning, explaining, and making sense of the mathematics that prospective teachers will teach is needed to produce well-started beginning teachers. Although the quality of mathematical preparation is more important than the quantity, the following recommendations are made for the amount of mathematics coursework for prospective teachers.
    I. Prospective elementary teachers should be required to complete at least 12 semester-hours on fundamental ideas of elementary mathematics, their early childhood precursors, and middle school successors.
    II. Prospective middle grades (5–8) teachers of mathematics should be required to complete at least 24 semester-hours of mathematics that includes at least 15 semester-hours on fundamental ideas of school mathematics ap- propriate for middle grades teachers.
    III. Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in mathematics that includes three courses with a primary focus on high school mathematics from an advanced viewpoint.

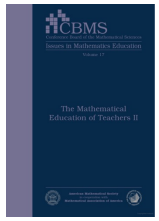
(Conference Board of the Mathematical Sciences, 2012)



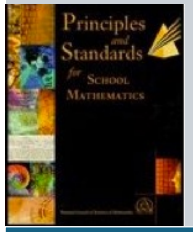
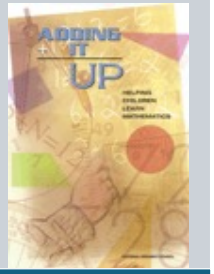
The Mathematical Education of Teachers II

- Recommendation 3. Throughout their careers, teachers need opportunities for continued professional growth in their mathematical knowledge.
- Recommendation 4. All courses and professional development experiences for mathematics teachers should develop the habits of mind of a mathematical thinker and problem-solver, such as reasoning and explaining, modeling, seeing structure, and generalizing. Courses should also use the flexible, interactive styles of teaching that will enable teachers to develop these habits of mind in their students.

(Conference Board of the Mathematical Sciences, 2012)



Mathematical Habits of Mind

NCTM Process Standards	Standards of Mathematical Practice	Proficiencies as Described in Adding It Up: Helping Children Learn Mathematics
• Problem Solving • Reasoning and Proof • Communication • Connections • Representations	• Make sense of problems and persevere in solving them. • Reason abstractly and quantitatively. • Construct viable arguments and critique the reasoning of others. • Model with mathematics. • Use appropriate tools strategically. • Attend to precision. • Look for and make use of structure. • Look for and express regularity in repeated reasoning.	• Strategic Competence • Conceptual Understanding • Procedural Fluency • Productive Disposition • Adaptive Reasoning
		
+ Habits of Mind (Cuoco, Goldenberg, and Mark, 1996)		

Statistical Education of Teachers

Recommendation 1. Prospective teachers need to learn statistics in ways that enable them to develop a deep conceptual understanding of the statistics they will teach.

Recommendation 2. Prospective teachers should engage in the statistical problem-solving process—formulate statistical questions, collect data, analyze data, and interpret results—regularly in their courses.

Recommendation 3. Because many currently practicing teachers did not have an opportunity to learn statistics during their pre-service preparation programs, robust professional development opportunities need to be developed for advancing in-service teachers' understanding of statistics.

Recommendation 4. All courses and professional development experiences for statistics teachers should allow them to develop the habits of mind of a statistical thinker and problem-solver, such as reasoning, explaining, modeling, seeing structure, and generalizing.

Roles for Teacher Educators in Statistics Recommendation 5. At institutions that prepare teachers or offer professional development, statistics teacher education must be recognized as an import-ant part of a department's mission and should be undertaken in collaboration with faculty from statistics education, mathematics education, statistics, and mathematics.

Recommendation 6. Statisticians should recognize the need for improving statistics teaching at all levels.

Franklin, C., Bargagliotti, A., Case, C., Kader, G., Schaeffer, R., Spangler, D. (2015. *The Statistical Education of Teachers*. American Statistical Association, http://www.amstat.org/education/SET/SET.pdf

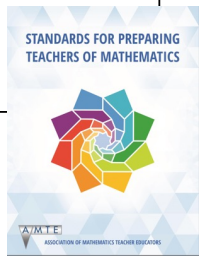
The Mathematical Education of Teachers III (In Progress)

- MET III will serve as a resource for those who teach mathematics—and statistics—to PreK–12 mathematics teachers, both future teachers and those who already teach in our nation's schools.
- The document will make recommendations for the mathematics that teachers should know and how they should obtain that knowledge.
- It will urge greater involvement of mathematicians and statisticians in teacher education so that the nation's mathematics teachers have the knowledge, skills, and dispositions needed to provide students with a twenty-first century mathematics education that ensures high school graduates are college- and career-ready.
- In addition to the mathematical sciences knowledge for teaching the book will focus on broader issues impacting teacher education, such as such as teacher certification policies, equity and access, STEM pathways, and recruitment and retention of a diverse pool of mathematics teachers.
- Furthermore, CBMS is excited to be working on the MET III at the same time as the American Statistical Association is working on the *Statistical Education of Teachers II (SET II), a revision of the ASA's SET document published in 2015*.
- MET III and SET II will have separate writing teams, with extensive plans for collaboration between the two teams

Standards for Preparing Teachers of Mathematics

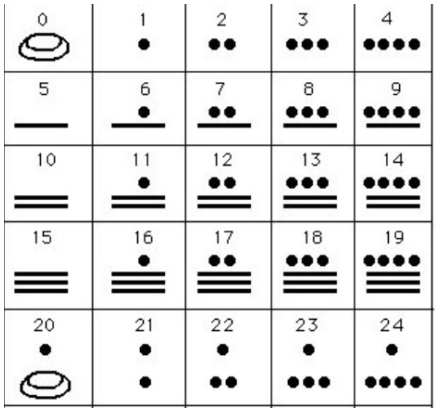
Standard	Indicators
C.1. Mathematics Concepts, Practices, and Curriculum Well-prepared beginning teachers of mathematics possess robust knowledge of mathematical and statistical concepts that underlie what they encounter in teaching. They engage in appropriate mathematical and statistical practices and support their students in doing the same. They can read, analyze, and discuss curriculum, assessment, and standards documents as well as students' mathematical productions.	C.1.1 Know Relevant Mathematical Content C.1.2 Demonstrate Mathematical Practices and Processes C.1.3. Exhibit Productive Mathematical Dispositions C.1.4. Analyze the Mathematical Content of Curriculum C.1.5 Analyze Mathematical Thinking C.1.6 Use Mathematical Tools and Technology

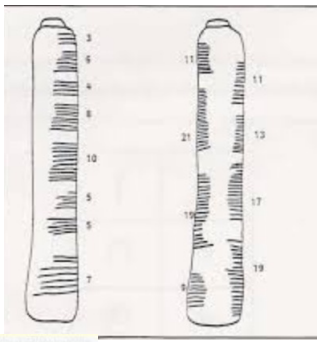
(Association of Mathematics Teacher Educators, 2017)

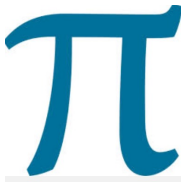


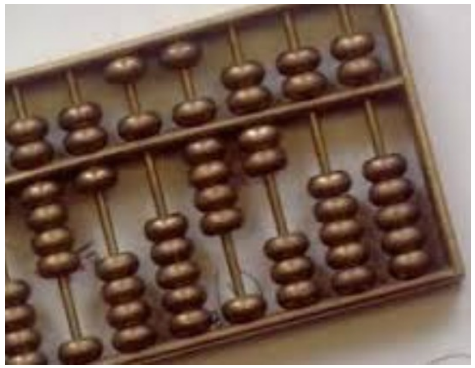
Mathematical Experiences and Knowledge Teachers Need

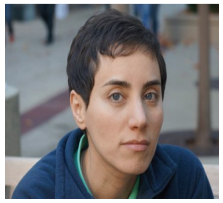
Historical











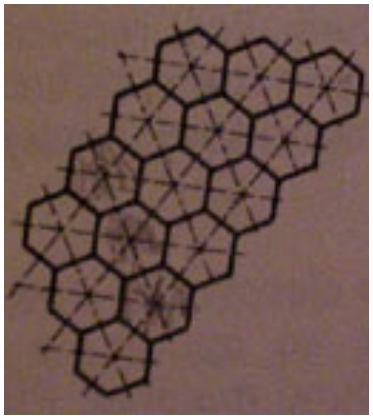
Ethnomathematics

Mathematics which is practiced among identifiable cultural groups:

- National-tribal societies
- Labor groups
- Children of a certain age bracket
- Professional classes

Its identity depends largely on focuses of interest, on motivation, and on certain codes and jargons which do not belong to the realm of academic mathematics

(D'Ambrosio,1997, p. 16).

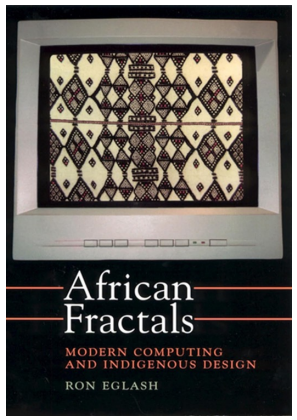


Weiger, P. R. (2000). Re-Calculating math instruction. Black Issues in Higher Education,17(13), 58 – 63.

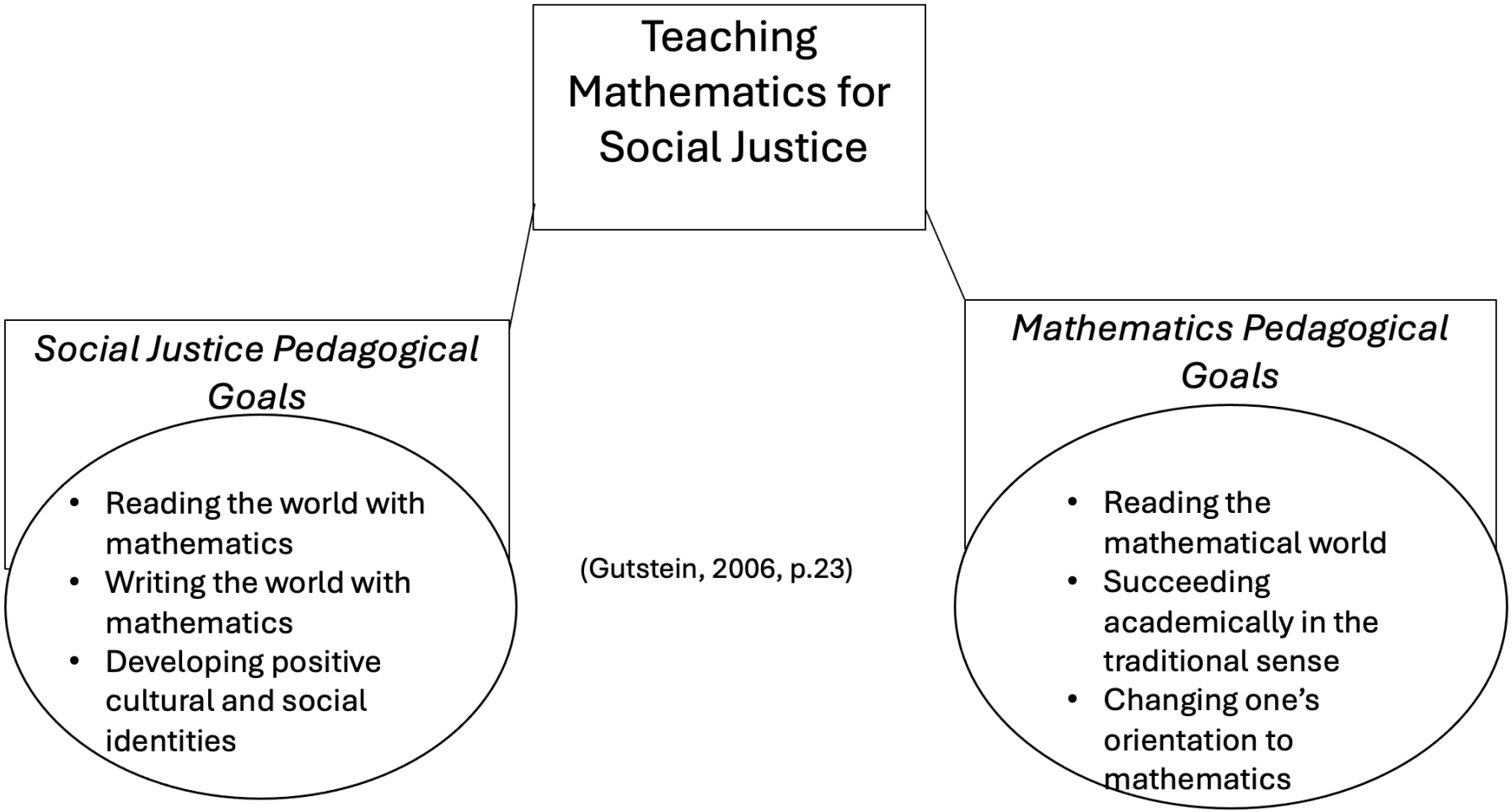
Other Examples of Ethnomathematics

- The examination of ratios, patterns and symmetry in Japanese origami;
- Logic of kin relations (e.g., Warlpiri in Australia);
- Chance and strategy games and puzzles from various Native American tribes;
- Symmetry and concepts of impermanence in the mandalas of the East;
- Measuring and ratios in traditional quilting patterns (Presmeg, 1998);
- Counting and understanding of time-keeping in the pagan Misseri Calendar, which was created by Icelanders who were greatly influenced by their environment (Bjamadottir, 2010);
- Fractals in African design (Eglash, 2007);
- Shapes and design in graffiti from hip-hop culture (Eglash, 2012).

(Brandt & Chernoff, 2014)



Other Examples:


Teaching Mathematics for Social Justice
Social Justice Pedagogical Goals
Reading the world with mathematics
Writing the world with mathematics
Developing positive cultural and social identities
(Gutstein, 2006, p.23)
Mathematics Pedagogical Goals
Reading the mathematical world
Succeeding academically in the traditional sense
Changing one's orientation to mathematics

Teaching Mathematics for Social Justice

- Driving While Black or Brown
- Food Deserts
- Social Justice Lesson Related to Disabilities and Access
- Gerrymandering
- Mathematics Related Health Disparities
- Mathematics Related to Students Opportunities to Learn
- Fair Housing Issues

In 2022, Ladson-Billings defined culturally relevant teaching as being comprised of three components which focus on:

1. Ensuring that student learning is taking place.
2. Developing student cultural competence --the ability of students to draw on their own backgrounds, languages, histories, customs, and experiences as they gain fluency and facility in at least one other culture.
3. Fostering student critical consciousness through which they challenge the status quo of the current social order.

(Will, 2022)

Students need opportunities to see themselves in the curriculum (mirror) as well as have a view of others in the world (window) (Gutiérrez , 2007, 2018).

Cultural Competence- Firm grounding in one's culture of origin while acquiring fluency in a least one more culture (Ladson-Billings, 2017).

Culturally Relevant and Mathematical Modeling

The Task:

It is important to provide an appropriate amount of time between classes. The school wants to make sure that each student can quickly and safely arrive at their next class. However, next year, the school's population will increase from 630 to 830. Can the school's current time between classes remain the same with this increase in population? Based on the information in the handouts, determine how much time should be allowed between classes once the high schools combine. Be able to justify your estimate of time.

Secondary Mathematics Education Content Course Requirements at Auburn University

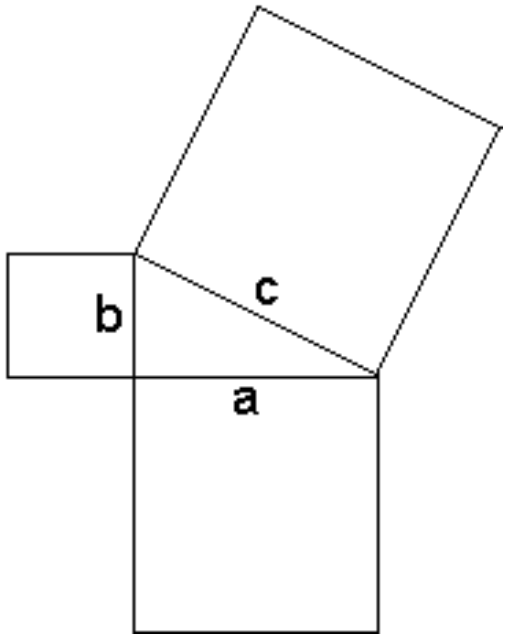
- Calculus I
- Calculus II
- Calculus III
- History of Mathematics
- Linear Differential Equations
- Introduction to Advanced Mathematics
- Topics in Linear Algebra
- Introduction to Abstract Algebra I
- Intermediate Euclidean Geometry I
- Probability and Statistics I
- Discrete Math Elective
- Analysis I
- A 5000 level Course

Courses More Focused on the Mathematical Knowledge for Teaching and Active Learning

- The Mathematics of Doing, Understanding, Learning and Educating for Secondary Schools (MODULE(S2)) project creates and supports the use of undergraduate mathematics course materials that provide opportunities for users to develop deep mathematical knowledge of ***geometry***, ***statistics***, ***algebra***, and **modeling**, specifically as it relates to doing the work of teaching.
- Student Engagement in Mathematics through an Institutional Network for Active Learning (SEMINAL)
  - Created models for a national push to reform math instruction.
  - Studied and scaled the implementation of active learning to dramatically increase student success in introductory math courses, which are foundational to the study of STEM fields.
  - SEMINAL handbook published in 2021 helps mathematics departments improve student outcomes.

Grew out of the work of the Mathematics Teacher Education Partnership

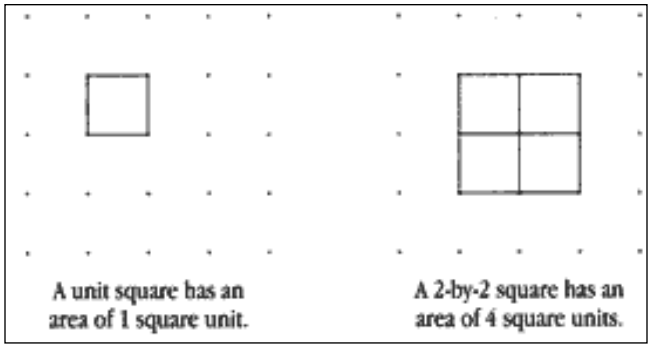
Let’s look at the Pythagorean Theorem across mathematical contexts.



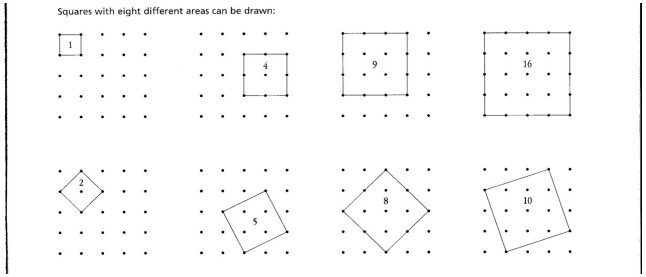
Eighth Grade Mathematics

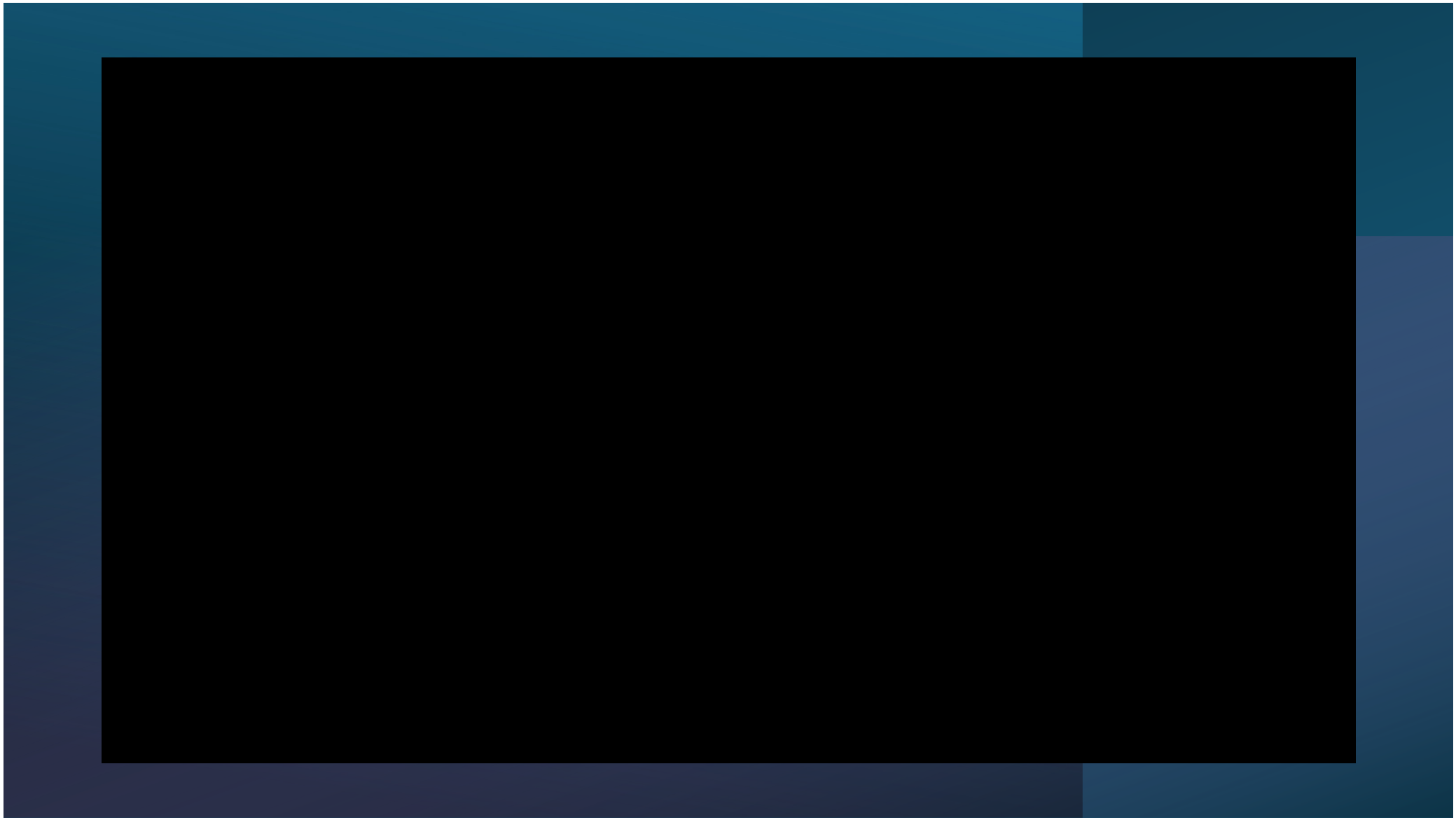
- Develop strategies for finding the distance between two points on a coordinate grid
- Explain a proof of the Pythagorean Theorem
- Understand and use the Pythagorean Theorem to solve everyday problems
- Write fractions as repeating or terminating decimals
- Write decimals as fractions
- Recognize rational and irrational numbers
- Locate irrational numbers on a number line
- Relate the area of a square to its side length, and the volume of a cube to its side length
- Estimate square roots and cube roots

Sample Activity

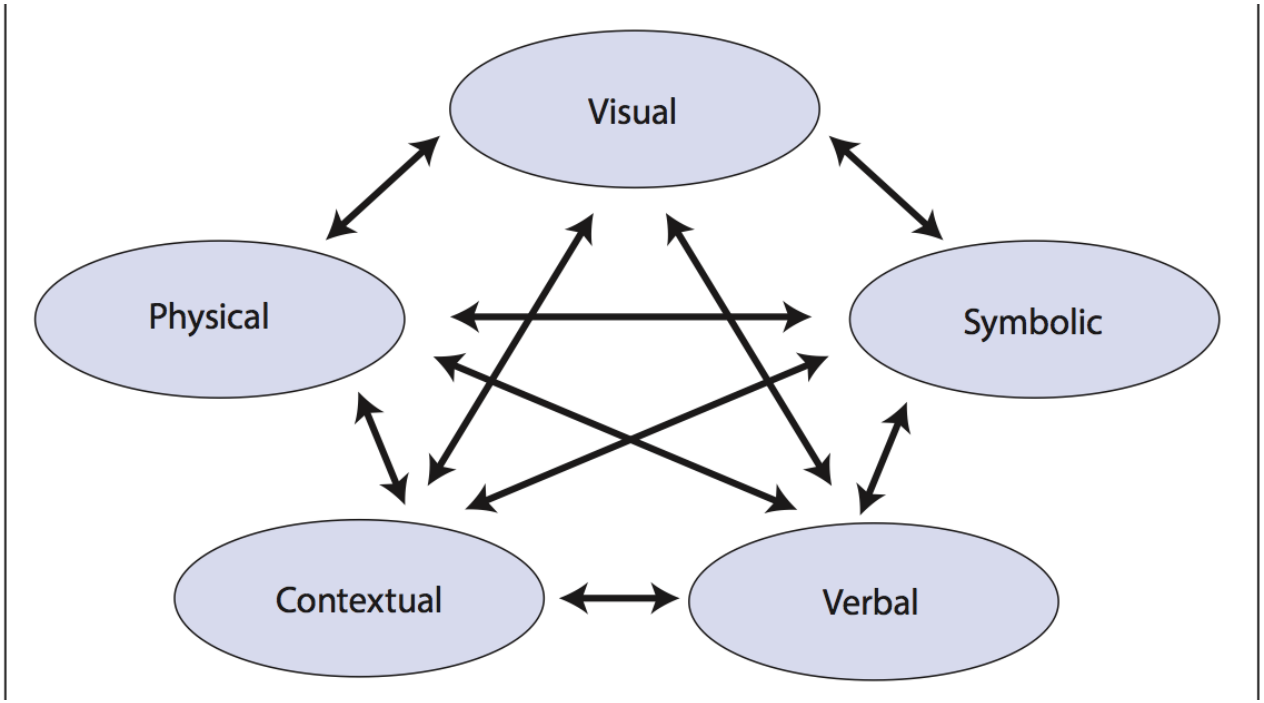


On the 5-dot-by-5-dot grids on Lab sheet 2.2, draw squares of various sizes by connecting dots. Try to draw squares with as many different areas as possible. Label each square with its area.





Kinds of Representations (NCTM, 2014, p. 25)

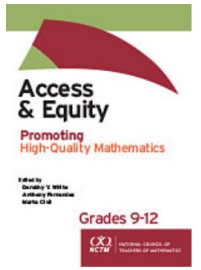


Historical

The most famous Babylonian mathematical tablet, Plimpton 322, dated between 1820 BCE and 1762 BCE, contains sophisticated mathematics bridging between geometry and algebra and touching on results that were believed to have been discovered during the Hellenistic period, hundreds of years later. There is evidence that many ancient civilizations were aware of the result we know as the Pythagorean theorem, $c^2 = a^2 + b^2$, where c is the hypotenuse of a right triangle, and a and b are the two legs. Because this theorem is attributed to Pythagoras, any three positive integers a, b, and c that satisfy this equation are called a Pythagorean triple. A formula for generating such triples was discovered during Hellenistic times by the most original mathematician of late antiquity, Diophantus of Alexandria (ca. 250 CE). In 1945, Neugebauer and Sachs decoded the Babylonian clay tablet Plimpton 322. The text involves a table with four columns of numbers, which after careful scrutiny turned out to yield Pythagorean triples.

(Glaz, 2020, p. 41)

Disability, Accessibility, and the Pythagorean Theorem: One Teacher's Approach to Teaching for Equity And Social Justice



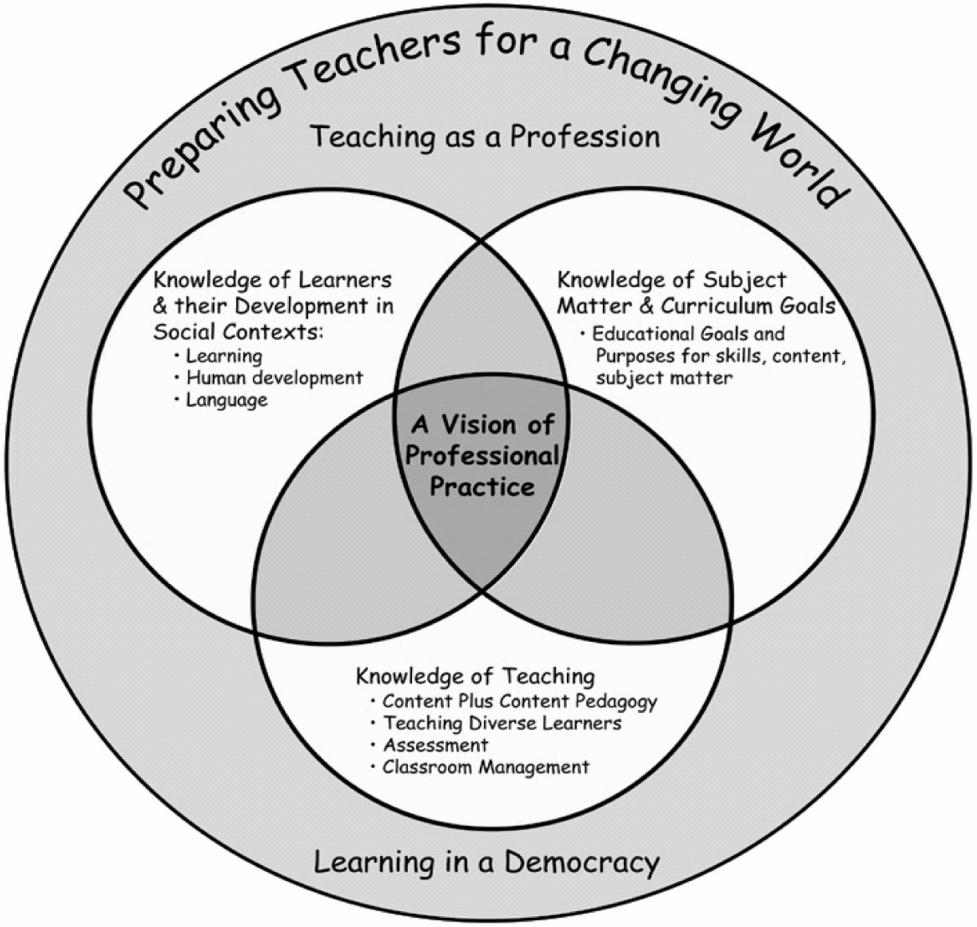
- **Tasks and questions given to students:**
  - Write down everything you know about the Pythagorean theorem.
  - Define disability.
  - Define accessibility.
  - How can we increase accessibility at our school?
  - How many students come to our school in a wheelchair?
  - How is the Pythagorean Theorem useful in everyday life?
- **Developed into a social justice lesson related to disabilities and access**

(Maloney,, Murray, and Rudin, (2018)

Other uses for the Pythagorean Theorem: Used in construction and architecture. Used in two-dimensional navigation to find the shortest distance. Used to survey the steepness of the slopes of mountains or hills.

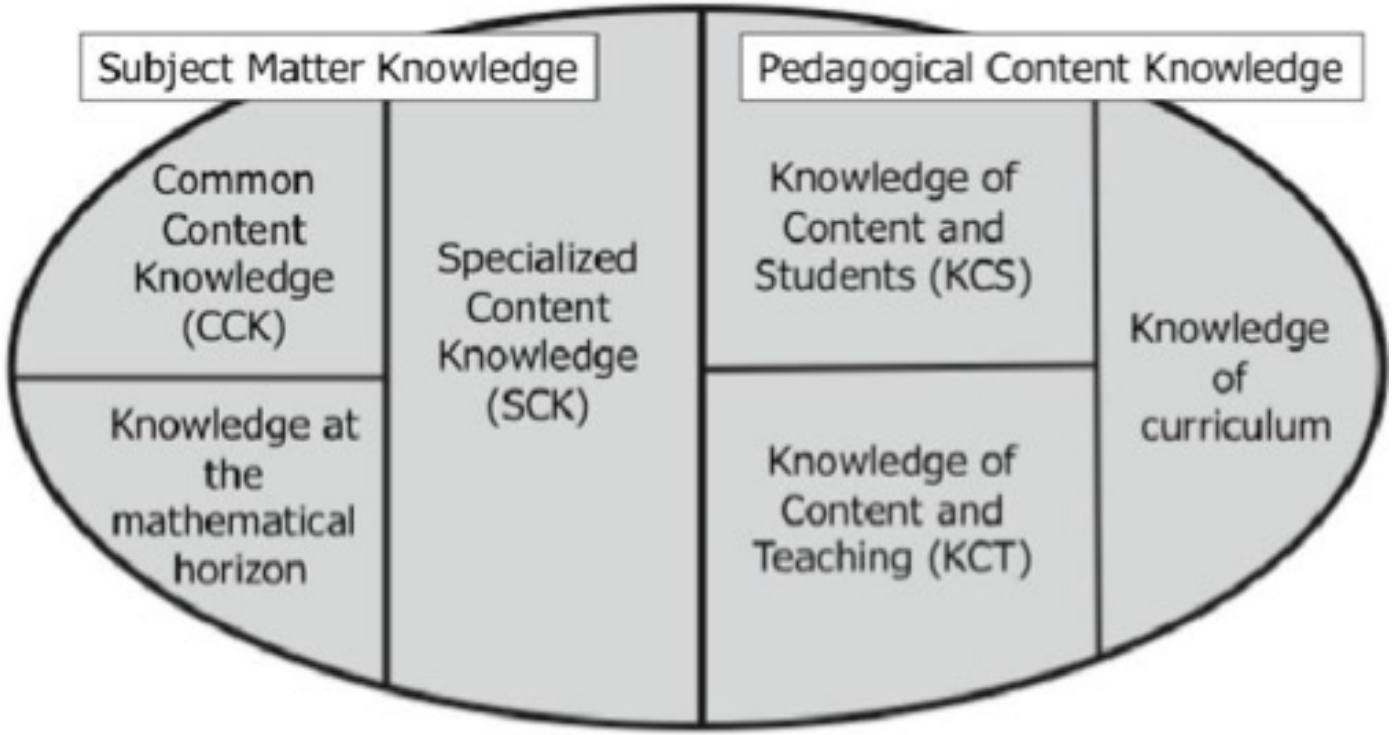
Classic Proofs

../Extending Two Classic Proofs of the Pythagorean Theorem to the Law of Cosines.pdf



(Darling-Hammond & Bransford, 2005, p. 11; as cited in Darling-Hammond, 2006)

Framework for the 'Domains' of Mathematical Knowledge for Teaching



(Ball, Thames, & Phelps, 2008)